Fractional solution for the non-Newtonian MHD blood flow in the porous artery

Salah Uddin*, Mahathir Mohamad*, Mahmod Abd Hakim Mohmad†, Rozaini Roslan*

Abstract

In the present study, third-grade magneto-hydrodynamic (MHD) blood flow for the non-Newtonian type fluid inside porous channel accelerated by the periodic pressure gradient is analytically simulated with the rational order derivative. Motivated from the recent advancement in the fractional calculus non-linear governing equations are coupled with CF (Caputo-Fabrizio) time fractional order derivative with initial and boundary conditions. Analytical time fractional expressions for velocity are also presented. Moreover, the imposed body acceleration is considered with Laplace and finite Hankel transforms.

Keywords: CF derivative, Hankel transform, pulsatile flow.

Introduction

Human blood contains RBC's, white blood cells, plasma and important nutrients like glucose and mineral ions. Red blood Cells (RBC's) are iron and oxygen carrier particles, consequently the performance is intense to the external magnetic field. Effective and potential applications on the non-local character, viscoelasticity and non-linearity are presented by researchers nowadays.

To study fluid transportation, two-dimensional steady magneto hydrodynamics flow over a permeable surface with convective heating boundary conditions was presented by Yazdi *et al.* (2011). The study reflects that third-grade parameter and magnetic field decrease the fluid velocity Adesanya and Falade (2015). Small arteries show high flow resistance than as compared to the large arteries Mekheimer and Kot (2015).

From the graphical analysis, it is revealed that fall in thermal diffusivity decelerate the fluid flow and hence mass flux gets reduced Ahmed and Dutta (2015). A new definition of fractional order derivative with a smooth kernel was presented Caputo and Fabrizio (2015). Numerical study of magneto-hydrodynamic blood flow inside arteries and capillaries was studied Akbarzadeh (2016). A time-dependent stretching velocity of the unsteady coupled flow along with surface temperature was theoretically analyzed Sinha, Misra and Shit (2016). Computational results reveal that unsteadiness parameter

*Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia, Pagoh, Johor, Malaysia. salahuddinjalil@gmail.com

†Faculty of Diploma Study Centre, Department of Mechanical Engineering, Universiti Tun Hussein Onn Malaysia, Pagoh Educational Hub,84600 Pagoh, Johor, Malaysia

enhanced the rate of heat transfer. Numerically simulated the healthy, diabetic and anemic models by using finite volume method Alshare and Tashtoush (2016). Results show that wall shear stress was reduced by 1.6 times for twice arterial diameter. Finite difference method along with L1-algorithm was used to study the unsteady Maxwell flow over a boundary layer moving plate, where convection flow was enhanced with temperature relaxation time Cao *et al.* (2016).

Theoretical and numerical analysis was performed to analyze the velocity and temperature distributions Jhankal, Jat and Kumar (2017). They have studied different values of the unsteady parameter A and transverse magnetic field. Furthermore, skin-friction coefficient and Nusselt number were also presented numerically. It was revealed that the unsteadiness parameter enhanced the rate of heat transfer. Partial slip model of stabilized approximation indicates that velocity increases with third grade and viscoelastic modulus Rasheed *et al.* (2017). On the other hand, it is a decreasing function of cross-flow velocity. Role of non-integer fractional derivative along with other embedded parameters on fluid velocity, temperature, and mass concentration distribution was studied Khan *et al.* (2017), Shahid (2015) and Uddin *et al.* (2018). The physical advantage of the fractional model was noticed for small and large time. In addition, velocity and temperature were decreasing with the spatial variable and Prandtl number. Moreover, Schmidt number negatively influenced the mass concentration of fluid flow over a vertical oscillating plate. The effect of CF fractional order derivative is studied in the case of Newtonian and non-Newtonian incompressible flow problems Uddin *et al.* (2019).

In the present paper, CF (Caputo-Fabrizio) fractional MHD (magneto-hydrodynamic) blood flow inside porous artery with pulsatile nature is studied. Non-Newtonian third-grade blood flow under the action of periodic body acceleration along with periodic pressure gradient is considered. CF rational order derivative is analytically solved by first considering Laplace transform and then applying the finite Hankel transform.

Formation of the Problem

Pulsatile laminar flow with unsteady behavior for non-Newtonian third grade incompressible human blood inside the porous artery is investigated. A schematic diagram in the cylindrical coordinate systems (r, θ, z) describing the mechanical physics of the blood flow model is shown in Figure. 1. Non-Newtonian third grade blood flows through a fully porous artery in z-direction of radius R. At the outer wall velocity slip condition is assumed.

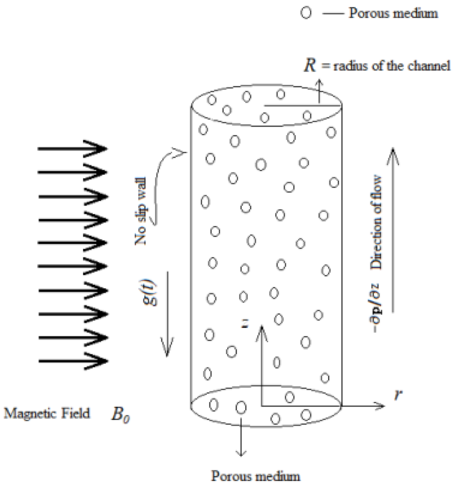

Figure1: MHD pulsatile blood flow in a porous artery

Pumping action of heart produces pressure gradient $-\frac{\partial p}{\partial z}$ which stimulates the pulsatile blood flow

$$-\frac{\partial p}{\partial z}=A_0+A_1\cos(\omega_p t), \qquad (1)$$

where in Eq. (1), A_0 and A_1 are steady part and amplitude of the pressure fluctuation responsible for systolic and diastolic pressures.

In most of the cases sudden movement during exercise or passengers sitting in a fast speed train or in a vehicle, human body as well as insides blood resisted by the external acceleration and gravitational field. Normal blood flow inside human arteries is disturbed under certain conditions. Which may lead to various physiological problems like pulse abnormality, vision problem, headache and so on.

$$g(t)=A_g\cos(\omega_g t+\vartheta). \qquad (2)$$

Therefore, body acceleration is assumed in Eq. (2) due to its physiological importance. Where ω_p is the frequency of the heart pressure. Furthermore, $g(t)$ represents the body acceleration, A_g is the acceleration amplitude, ω_g is the frequency, and body acceleration makes an ϑ with the pressure gradient. Gravity forces are ignored. Governing flow phenomenon in the axial path is [6]:

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\{div\,\tau\}_z-\frac{\mu}{K}u+\rho A_g\cos(\omega_g t+\vartheta)-\sigma B_o{}^2 u. \qquad (3)$$

The non-dimensional variables are,

$$r'=\frac{r}{R}, u=\frac{u}{U\infty}, t=\frac{\omega_p t}{2\pi}. \qquad (4)$$

The CF non-dimensional model of the momentum equations along with the initial and boundary conditions becomes,

$$\varsigma^2 D^\alpha u = B_1(1+\gamma\cos(2\pi t)) + B_2(\cos(2\pi\omega t)+\vartheta) + \left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) +$$

$$\Lambda\left\{\left(\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)^3 + 3\frac{\partial^2 u}{\partial r^2}\left(\frac{\partial u}{\partial r}\right)^2\right)\right\} - (M^2+P)u, \qquad (5)$$

$$u = -\frac{B_1+B_2}{M^2+P}\left\{1 - \frac{I_0(\sqrt{M^2+P}\,r)}{I_0(\sqrt{M^2+P})}\right\} \text{at t} = 0, \qquad (6)$$

$$\frac{\partial u}{\partial r} = 0 \,\text{at r} = 0, \qquad (7)$$

$$u = 0 \,\text{at r} = 1, \qquad (8)$$

where I_0 is a modified Bessel function of first kind.

Solution to the Problem

The recent advancement in the fractional calculus named as CF rational order derivative works with the temporal variable *t* and spatial variable *r*. In the first case, it is significant to proceed with a Laplace transform (*LT*). Therefore, applying the *LT* with respect to time *t* in Eq. (7), utilizing the initial and boundary conditions given in Eq. (9), we obtain,

$$\varsigma^2 \frac{s\bar{u} - u|_{t=0}}{s+(1-s)\alpha} = B_1\left(\frac{1}{s} + \gamma\frac{s}{s^2+4\pi^2}\right) + B_2\left(\frac{s\cos\vartheta - 2\pi\omega\sin\vartheta}{s^2+4\pi^2\omega^2}\right) +$$

$$\left(\frac{1}{r}\frac{\partial\bar{u}}{\partial r} + \frac{\partial^2\bar{u}}{\partial r^2}\right) + \Lambda\left\{\left(\frac{1}{r}\left(\frac{\partial\bar{u}}{\partial r}\right)^3 + 3\frac{\partial^2\bar{u}}{\partial r^2}\left(\frac{\partial\bar{u}}{\partial r}\right)^2\right)\right\} - (M^2+P)\bar{u}, \qquad (9)$$

$$\bar{u} = -\frac{B_1+B_2}{M^2+P}\left\{1 - \frac{I_0(\sqrt{M^2+P}\,r)}{I_0(\sqrt{M^2+P})}\right\} \text{at t} = 0, \qquad (10)$$

$$\frac{\partial\bar{u}}{\partial r} = 0 \,\text{at r} = 0, \quad (11)$$

$$\bar{u} = 0 \,\text{at r} = 1, \quad (12)$$

where $\bar{u} = L(u)$. Further taking the finite Hankel transform. Eq. (10) with conditions stated in Eq. (11)

$$\bar{u}_H\left(\varsigma^2\frac{s}{s+(1-s)\alpha} + r_n^{\,2}(\Lambda+1) + M^2 + P\right) = \frac{J_1(r_n)}{r_n}\left[B_1\left(\frac{1}{s} + \gamma\frac{s}{s^2+4\pi^2}\right) +\right.$$

$$\left. B_2\left(\frac{s\cos\vartheta - 2\pi\omega\sin\vartheta}{s^2+4\pi^2\omega^2}\right) + \varsigma^2\frac{s}{s+(1-s)\alpha} \times \frac{B_1+B_2}{\Lambda(r_n^{\,2}+M^2+P)}\right]. \qquad (13)$$

Axial velocity I

To obtain the axial velocity of the flow model in the porous artery, inverse Laplace and Hankel transformations (L^{-1} and H^{-1}) of Eq. (14) involving Bessel functions of first kind were taken by coding in Mathematica.

$$u = 2\sum_{n=1}^{\infty} \frac{J_0(r\,r_n)}{J_1^{\,2}(r_n)} u_H . \qquad (14)$$

The axial flow velocity against various flow parameters of interest can be obtained via Eq. (14). The parameters used in the execution of simulated results are as follows:

$$a_{n1} = (\Lambda+1)r_n^{\,2} + M^2 + P, a_{n2} = \frac{B_1 + B_2}{\Lambda(r_n^{\,2} + M^2 + P)} . \qquad (15)$$

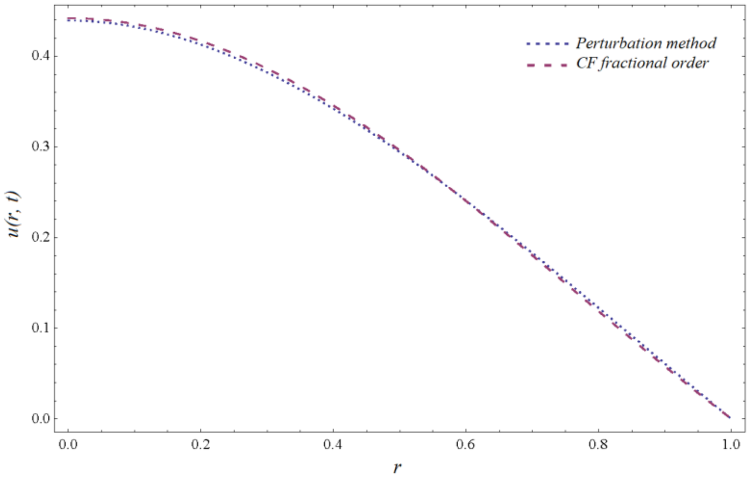

Figure 2: Velocity comparison between Akbarzadeh (2016) and the CF model

Conclusion

Pulsatile blood flow affected by the external magnetic field is simulated. CF rational order derivative is also applied to the non-Newtonian model. Moreover, the blood is modelled as the third-grade fluid with initial and boundary conditions (fixed initial and boundary conditions). CF rational order derivative is analytically calculated with *LT* and then applying the finite Hankel transform. Governing Navier-stokes equations are solved against both local and non-local models. Results are agreed well with the previously reported findings. It is expected that the study will be supportive in the medical and experimental examinations of various arterial infections.

References

Adesanya, S. O., & Falade, J. A. (2015). Thermodynamics analysis of hydromagnetic third grade fluid flow through a channel filled with porous medium. *Alexandria Engineering Journal*, 54, 615-622.

Ahmed, N., & Dutta, M. (2015). Heat transfer in an unsteady MHD flow through an infinite annulus with radiation. *Boundary value problems*, 1-17.

Akbarzadeh, P. (2016). Pulsatile magneto-hydrodynamic blood flows through porous blood vessels using a third grade non-Newtonian fluids model. *Computer methods and programs in biomedicine*, 126, 3-19.

Alshare, A., & Tashtoush, B. (2016). Simulations of Magneto-hemodynamics in Stenosed Arteries in Diabetic or Anemic Models. *Computational and Mathematical Methods in Medicine*, 1-13.

Cao, Z., Zhao, J., Wang, Z., Liud, F., & Zheng, L. (2016). MHD flow and heat transfer of fractional Maxwell viscoelastic nanofluid over a moving plate. *Journal of Molecular Liquids*, 222, 1121-1127.

Caputo, M. & Fabrizio, M. (2015). A new Definition of Fractional Derivative without Singular Kernel. *Progress in Fractional Differentiation and Application. An international journal*, 2, 73-85.

Jhankal, A. K., Jat, R. N., & Kumar, D. (2017). Unsteady MHD Flow and Heat Transfer over a Porous Stretching Plate. *International Journal of Computational and Applied Mathematics*, 12, 325-333.

Khan, I., Ahmad, M., Shah, N. A., & Imran, M. A. (2017). Effects of Non-integer Order Time Fractional Derivative on Coupled Heat and Mass Transfer of MHD Viscous Fluid over an Infinite Inclined Plane with Heat Absorption. *International Journal of Innovative Research in Science, Engineering and Technology*, 6, 1-11.

Mekheimer, Kh. S. & Kot, M. A. El. (2015). Suspension model for blood flow through catheterized curved artery with time-variant overlapping stenosis. *Engineering Science and Technology. an International Journal*, 452-462.

Rasheed, A., Kausar, A., Wahab, A., & Akbar, T. (2017). Stabilized approximation of steady flow of third grade fluid in presence of partial slip. *Results in Physics*, 7, 3181-3189.

Shahid, N. (2015). A study of heat and mass transfer in a fractional MHD flow over an infinite oscillating plate. *Springer Plus*, 4, 1-21.

Sinha, A. Misra, J. C., & Shit, G. C. (2016). Effect of heat transfer on unsteady MHD flow of blood in a permeable vessel in the presence of non-uniform heat source. *Alexandria Engineering Journal*, 55, 2023-2033.

Uddin, S., Mohamad, M., Gorji, M. R., Roslan, R., Ibrahim, M., & Alarifi, I. M., (2019). Fractional electro-magneto transport of blood modeled with magnetic particles in cylindrical tube without singular kernel. *Microsystem Technologies*, 1-10.

Uddin, S., Mohamad, M., Kamardan, M. G., Sufahani, S., Mehmood, O. U., & Roslan, R. (2018). Fractional derivative on MHD flow of the third grade fluid in the porous tube subjected to a constant body acceleration and pressure gradient. *A Letter on Applications of Mathematics and Statistics*, 119-127.

Uddin, S., Mohamad, M., Khalid, K., Abdulhammed, M., Rusiman, M. S., Che-Him, N., & Roslan, R. (2018). Blood flow problem in the presence of magnetic particles through a circular cylinder using Caputo-Fabrizio fractionalderivative. *IOP Conf. Series: Journal of Physics: Conf. Series*, 995, 1-9.

Uddin, S., Mohamad, M., Mohamad, M. A. H., Sufahani, S., Kamardan, M. G., Mehmood, O. U., Wahid, F., & Roslan, R. (2018). Caputo-Fabrizio Time Fractional Derivative Applied to Visco Elastic MHD Fluid Flow in the Porous Medium. *International Journal of Engineering & Technology*, 7, 533-537.

Uddin, S., Mohamad, M., Sufahani, S., Kamardan, M. G., Mehmood, O. U., Wahid, F., & Roslan, R. (2018). Application of Caputo-Fabrizio Fractional Order Derivative (NFDt) in Simulating the MHD Flow of the Third Grade non-Newtonian Fluid in the Porous Artery. *International Journal of Engineering & Technology*, 7, 527-532.

Yazdi, M. H., Abdullah, S., Hashim I., & Sopian, K. (2011). Effects of Viscous Dissi-pation on the Slip MHD Flow and Heat Transfer past a Permeable Surface with Convective Boundary Conditions. *Energies*, 4, 2273-2294.