Numerical study of squeezing unsteady Fe_3O_4 and TiO_2-nanofluid flow with the existence of thermal radiation and heat generation /absorption

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Abstract

In this study, a numerical investigation of squeezing transient nanofluids flow between two parallel plates is considered. The impacts of radiant heating and thermal generation/absorption across the temperature profile were scrutinized. The Runge-Kutta fourth-order method (RK4) with shooting scheme was used to solve the two-dimensional non-linear momentum and energy equations. It is perceived that, incrementing the radiative parameter results in a decrease in the temperature profile, while the fluid thermal reading is vividly ascending in the cause of thermal generation but diminishes in the cause of thermal absorption respectively. The result shows a very good mutual agreement between the nano-sized Fe_3O_4 and TiO_2, though, a slight high upturn in thermal boundary layer thickness with TiO_2 is spotted than that of Fe_3O_4.

***Keywords*:** Parallel plates, Non-linear equations, ODEs, Runge-Kutta scheme (RK4)

Introduction

Substantial functioning thermal transport fluid models are the topic of numerous scrutinization in current decades. due to the esteem thermal conductivities' properties of metals, the thermal conductivity of suspensions of gritty solid in liquids have been an exciting research field for its enormous extent of physical instruments in many industrial and engineering processes. Many publications illustrate the present and prospective utilization of nanofluids Saleh, et al. (2017). Nano fluid has diverse applications in engineering and geophysical sciences. Among

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these comprises cooling of nuclear reactors, cooling of electronic equipment, and solar energy collectors, etc.

The flow of fluid and thermal transport via an unbounded upright plate has vast applications in science and technology. Amongst which involve packed-bed storehouse tanks and catalytic reactors etc. Lai et al. (1991) have analyzed the problem of nanofluid flow across an upward plate in the proximity of time exponential temperature on the boundary, their result shows that the nano particle concentration improves for fluids with little thermal conductivity while the fluid temperature is reducing.

Sheikholeslami et al. (2013) intensively carried out their research in relation to the squeezed unstable fluid flow using fourth-order Runge-Kutta (RK4). Nonetheless, some other fascinating and associated study is detailed in Hamza (1991). A scintillating research on the radiant heating was conducted in Kandasamy et al. (2013). Radiations perform an important task in heat transfer, as such, many researchers have carried out pieces of research on it. Hayat et al. (2013) have presented radiant heating impact in squeezing flows of Jeffery fluids. Alsaedi et al. (2012) studied the thermal generation/absorption impact within the standstill fluid flow. Hussaini et al. (2018) studied Bioconvection model for squeezing flow within parallel plates carrying gyrotactic microorganisms with radiative as well as thermal generation/absorption influences.

Moreover, the impacts of thermal generation/absorption were sufficiently examined and reported by Srinivasa and Eswara (2016), Abdel-wahed et al. (2015) and Madaki et al. (2018), respectively.

The goal of this study is to use Runge-Kutta fourth order scheme (RK4) along with shooting method to determine the numerical solution of squeezing unsteady Fe_3O_4 and TiO_2-nanofluid flow alongside radiant heating and thermal generation/absorption. The heating radiant and thermal generation/absorption impacts were analyzed within the temperature profile. Conversely, the numerical result obtained for lacking parameters $f'(0), f'''(0), \theta(0)$ for Fe_3O_4 were compared with previous result obtain by Madaki et al. (2018) with Cu-water.

Description of the Problem

The behavior of unstable nanofluid squeezed within two evenly spaced plates were perceived in this research. The distance within the two plates at any dimensionless time t is stated as $\omega = \pm\, l(1-\alpha t)^{\frac{1}{2}} = \pm h(t)$ where w is the distance, α is constant, and l is initial position (at $t = 0$). The flow model is deliberated along x and y axis and z is the axial coordinate which is studied to be zero from the flow province, the viscous distribution influence and thermal source due to friction were fastened.

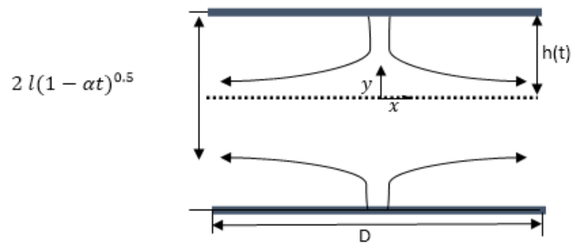

Fig 1: geometric configuration of the problem

The Runge-Kutta fourth order (RK4) with shooting scheme will be utilized on the modified nonlinear ODEs to examine the impression of both radiant heating and the thermal generation/absorption along with other parameter on the estimated solution of unsteady squeezing of Fe_3O_4 and TiO_2- nanofluid flow within the two parallel plates. Hence the fluid is nanofluid which comprised both ferrofluid and TiO_2.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{p_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{p_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = -\frac{1}{p_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{p_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$\frac{\partial T}{\partial y} + v\frac{\partial T}{\partial x} + u\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{Q(T - T_\infty)}{\rho_{nf}}, \tag{4}$$

where u and v serve as the velocities in x and y guidance, sequentially. whilst $p, T, T_{\infty}, f, \rho$ and Q are the pressure, the fluid temperature, an outline temperature, the fluid, the density and the thermal generation/absorption coefficient, respectively, and $\rho_{nf}, \mu_{nf}, (\rho C_p)_{nf}$ and K_{nf} are appropriately the effective density, dynamic viscosity, heat capacity, and thermal conductivity of the nanofluid as expressed by Domairry and Aziz (2009).

The Effective dynamic viscosity of nanofluid is given by

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}},$$

where φ is the solid volume fraction of nanoparticles. The effective density, ρ_{nf}, thermal diffusivity α_{nf} , and the heat capacitance of the nanofluid is provided as

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s,$$

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}},$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s,$$

The thermal conductivity of nanofluids confined to spherical nanoparticles is estimated by Max-well Garnett model

$$K_{nf} = \frac{k_f[k_s+2k_f-2\varphi(k_f-k_s)]}{[k_s+2k_f-\varphi(k_f-k_s)]}. \tag{5}$$

The subscripts nf, f, and s denote thermophysical properties of nanofluid, base fluid and nano-solid particles respectively.

The boundary conditions are assumed in the form:

$$v = v_w = \frac{dh}{dt},\ T = T_H \text{ at } y = h(t),$$

$$v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0 \text{ at} y = 0. \tag{6}$$

q_r is the radiation heat fluctuation, is express by Stefan-Boltzmann law as the radiant energy per unit time from a body is comparable to the fourth power of the absolute temperature i.e. $q_r = \sigma T^4 A$ where q is the heat transfer per unit time (ω), σ is the Stefan Boltzmann constant, T is the absolute temperature, A is the Area of the emitting body.

However, the radiant heat flux term appeared inside equation (4) was also vividly communicated by Roseland (1931) as,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{7}$$

Regarding some studies by Akbar et al. (2013), and Kothandapani and Prakash (2015) consider that the temperature disparity amid the flow is substantially confined, and the term T^4 can be weighed as a linear function of temperature. Hence, T^4 is broadened employing Taylor series expansion around T_∞ and neglecting the higher-order constants, we have

$$T^4 = T_\infty^4 + 4T_\infty^3 T - 4T_\infty^4,$$

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (8)$$

Substituting eqs. (7) and (8) into (4) gives

$$\frac{\partial T}{\partial y} + v\frac{\partial T}{\partial x} + u\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{k_{nf}}{(\rho C_p)_{nf}}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] - \frac{1}{\rho C_p}\frac{32\sigma^*}{3k^*}\frac{\partial T_\infty^3}{\partial y}\frac{\partial^2 T}{\partial^2} + \frac{Q(T-T_\infty)}{\rho_{nf}}. \quad (9)$$

Introducing the following non-dimensional quantities as in Madaki et al. (2018),

$$\omega = -\frac{\alpha x}{2l\left[\left((1-\alpha t)^{\frac{1}{2}}\right)\right]^3}f''(\eta),\ \eta = \frac{y}{\left[l(1-\alpha t)^{\frac{1}{2}}\right]},\ u = \frac{\alpha x}{[2(1-\alpha t)]}f'(\eta),\ v = -\frac{\alpha l}{[2(1-\alpha t)]}f(\eta),\ \theta(\eta) = \frac{T}{T_H},\ N = \frac{4\sigma^* T_\infty^3}{K_f \rho C_p K^*},\ \lambda = \frac{Q2l(T_w - T_\infty)(1-\alpha t)}{K_f \rho_f T_w}, A_1 = (1-\varphi) + \varphi\frac{\rho_s}{\rho_f},\ A_2 = (1-\varphi) + \varphi\frac{(\rho C_p)_s}{(\rho C_p)_f},$$
$$A_3 = \frac{K_{nf}}{K_f}, \quad (10)$$

And replacing the dimensionless quantities in equation (10) inside Eqs. (2) -(4) and getting rid of the pressure gradient from the summing equations (2) and (3) gives

$$f^{iv}(\eta) - SA_1(1-\varphi)^{2.5}\big(3f''(\eta) + \eta f'''(\eta) + f'(\eta)f''(\eta) - f(\eta)f'''(\eta)\big) = 0, \quad (11)$$

$$\left(-12A_3K_f + 16A_2N\right)\theta'' - 3p_r SA_2(f\theta' + \eta\theta') + \frac{3p_r Ec}{(1-\emptyset)^{2.5}}\left(f''^2 - 4\delta^2 f'^2\right) + 3\lambda A_2\theta = 0. \quad (12)$$

Using the following quantities as defined by Mustafa et al. (2012) S is the squeeze number, Pr and Ec are the Prandtl and Eckert integers, respectively

$$p_r = \frac{\mu_{f(\rho C_p)_f}}{p_f k_f}, S = \frac{\alpha l^2}{2v_f}, Ec = \frac{\rho_f}{(\rho C_p)_f}\left(\frac{\alpha x}{2(1-\alpha t)}\right)^2, \delta = \frac{1}{x}. \quad (13)$$

Equation (11) and (12) have to be solved subject to the following conditions

$f(0) = 0, f''(0) = 0, f(1) = 1, f'(1) = 0, \theta'(0) = 0, \theta(1) = 1.$ (14)

Method of solution with Runge-Kutta fourth order with shooting scheme

The boundary value problem of eq (11) and (12) with boundary condition (14) is solved using shooting technique by transforming it into an IVP. Therefore, we set

$f' = u,\ u' = f'' = v,\ v' = f''' = w,\ w' = f^{iv},\ \theta' = r(\eta),\ r'(\eta) = \theta''(\eta)$ (16)

With the boundary conditions

$$f(L) = 1,\ u(L) = 0,\ \theta(L) = 1 \quad (17)$$

using Runge-Kutta fourth order method, an appropriate estimate values for $f''(0), f'''(0), \theta(0)$ are formed until the boundary condition at infinity $f''(\infty), f'''(\infty), \theta(\infty)$ decay exponentially to zero. This calculation is performed with the help of shootlib function (a build-up function in Maple software). The effect of thermal radiation and heat generation/absorption in nanofluid flow squeeze through two parallel plate has been studied for different values of squeezing integer, radiation parameter, volume fraction of nano particles, temperature profile together with other salient parameters like the Prandtl number (Pr), Eckert number (Ec)

Results and discussion

This study involves the maturation of the mathematical model of the squeezed unstable nanofluid flow via the equidistant plates as manifested in Fig. 1, where the significant impact of the radiant heating and the thermal generation/absorption variables beside an appropriate parameter on the flow and heat transfer peculiarities have been analyzed numerically. We have used the Runge-Kutta fourth order with shooting scheme to solve the corresponding governing equations. However, the lacking functions $f'(0), f'''(0), \theta(0)$ have all been reckoned, where both the velocity and temperature profiles were analyzed and they were achieved numerically adopting RK4 scheme with shooting technique as $f'(0) = 1.42065576$, $f'''(0) = -2.06975916$, and $\theta(0) = 1.23192988$ respectively, With $S = 1, N = 0.5, Ec = 0.010, Pr = 6.20, \varphi = 0.020, \delta = 0.010, \lambda = 1.5$. This compared with the results obtained by Madaki et al. (2018) for (Cu-water) as $f'(0) = 1.41582016$, $f'''(0) = -2.01604099$, and $\theta(0) = 1.11883156$ respectively, With $S = 1, N = 0.5, Ec = 0.010, Pr = 6.20, \varphi = 0.020, \delta = 0.010, \lambda = 1.5$.

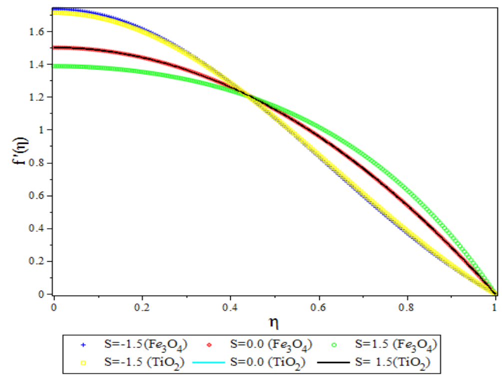

Figure 2: *The squeeze number influence on the velocity profile of the combined result with both Fe_3O_4 and TiO_2influence of squeeze number.*

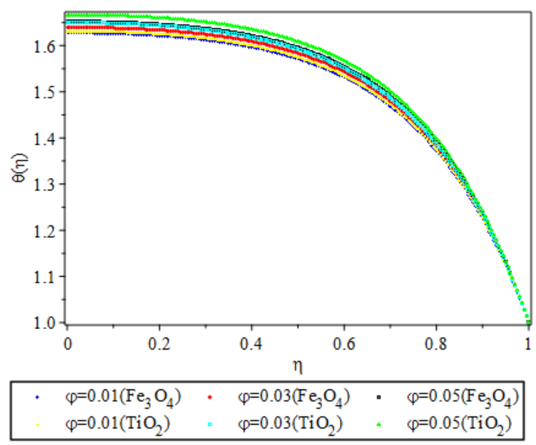

Figure 3: *The volume fraction influence on the temperature profile of the Combined results with both Fe_3O_4 and TiO_2.*

In figure 2., its observed that the squeeze number S, outlines the migration of the plates, i.e., when $S > 0$ it matches with the plates traveling independently, whilst $S < 0$ concurs with the plates progressing conjointly. Both (+ and -) squeezed characters have diverse influences on the velocity profile. However, it shows that both Fe_3O_4 and TiO_2 are mutually agreed as the plates move collectively. While **Fig.3** shows that when the magnitude of nano particles is added into the traditional fluid with $S = 1, Ec = 0.1, Pr = 6.2, \delta = 0.1, \lambda = 0.5, N = 0.5,$ there is growth in thermal boundary layer thickness. It is noticeable that on increasing the volume of the solid particle, the heat transfer is anticipated to have been improved on account of their great thermal conductivity. But due to the residence of radiant heating, the temperature goes up far away from the surface of the plates.

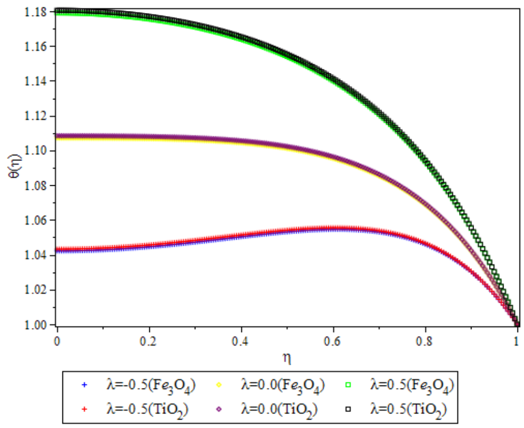

Figure 4: *The heat source influence on the $\theta(\eta)$ with Fe_3O_4 and TiO_2 combined.*

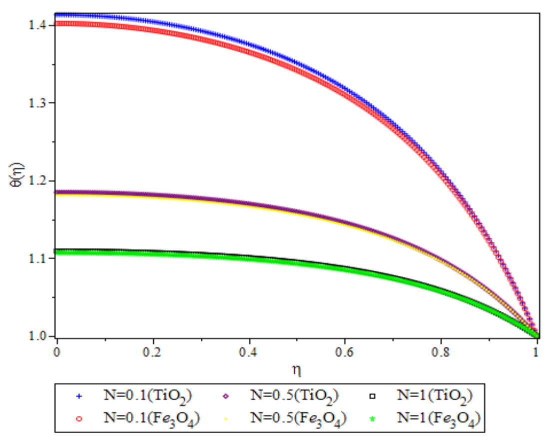

Figure 5: *The thermal radiation influence on the temperature profile with Fe_3O_4 and TiO_2 combined.*

Fig.4 exhibits the heat source influence on the temperature profile, it is observed that the fluid thermal reading is conspicuously ascending in the cause of thermal generation and declines in the cause of thermal absorption sequentially. It also shows the mutual agreement between Fe_3O_4 and TiO_2 with effect of heat source parameter. The temperature profile of nanofluid $\theta(\eta)$, is being influenced by thermal radiation parameter N for the fixed values of governing parameters, as portrayed in

Fig. 5, it is evident from this curves that thermal radiation frequently decelerates with slight increase in N. It is also observed that heat transfer coefficient continuously decreases, meaning, improving the radiant parameter leads to the reduction in the temperature profile. It also shows the mutual agreement between Fe_3O_4 and TiO_2, moreover, it is noticed that the temperature profile reduces slightly faster with Fe_3O_4 than TiO_2 as the radiation parameter is being increased.

Conclusions

The RK4 scheme along with shooting technique was employed on the modified nonlinear ODEs to investigate the impacts of both the radiant heating and the thermal generation/absorption on the estimated solution of squeezed unstable nanofluid flow within two equidistant plates. The water base fluid couple with Fe_3O_4 and TiO_2 as the solid nano particles were carefully used and the variation between the two materials utilized was recorded. The dimensionless velocity and temperature outlines have been studied. It has been observed that both negative and positive squeezed integers have peculiar significance influence on the velocity profile. Moreover, it is quite ostensible that the moment nano particles are being augmented into the fluid, the heat transfer is enhanced, due to their great thermal conductivity feature. Thus, the desired and fascinating results were achieved numerically. The influences of all the prominent parameters concerned have been reviewed and exhibited graphically.

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