Joint Image Bias Field Correction and Segmentation

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Abstract

It is a well-known fact that in modern technology, digital images play a vital role. Image processing is used in order to accurately use these images. The main purpose of image segmentation is to separate a particular image into regions or objects that are meaningful. But image segmentation becomes very challenging when there is intensity inhomogeneity or bias field in the given image's. Several models have been proposed in order to perform accurate segmentation but still there are issues. For this purpose we design a novel model for efficiently segmenting and bias field correction. Experimental results on medical, out-door and synthetic images validate the performance of the proposed model in contrast with the existing models.

Keywords: Bias Field Correction, Image Segmentation, Calculus of Variations, Level Set Method, Partial Differential Equations.

Introduction

Digital image processing is a computer-based technology that automates the processing, transformation, and display of visual data. It is becoming increasingly significant in many facets of our daily lives. It can also be found in a wide range of science and technology disciplines and fields, with applications in computerised photography, remote sensing, robotics, industrial inspection, medical diagnosis and auto-pilot planes, forensics, and the film industry, among others. For this reason, image processing often employed three mathematical methods to process an image probability and statistics methods, wavelet-based methods, and methods based on partial differential equations (PDEs) (Khan, Ali et al. 2020),(Ali, Sher et al. 2020). However, PDE-based approaches were primarily used. There are various theories and methods for solving PDEs in numerical computation, which is one advantage of using them in image processing.

Digital image processing is a technique in which an image or picture is transferred to a digital format and then computer processes are

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used to the image to produce an enhanced image or a virtually beneficial result (Tai, Lie et al. 2006), (Catté, Lions et al. 1992), (Mumford and Shah 1989), (Blomgren, Chan et al. 2000). Digital image processing is a collection of computational algorithms for evaluating, compressing, improving, and reconstructing images. This method employs digital computers to manipulate digital images. From image processing (Song and Zhang 2019) applications, there are some well known branches of image processing task namely such as de-noising (Chan and Chen 2006), (Chang and Chern 2003) debluring, image enhancement, image recognizing, image segmentation (Lie, Lysaker et al. 2006) and image bias field correction. Here our main concern is with image bias field correction and image segmentation task. The aim of segmentation is to partitioning the given image into finite numbers of meaningful regions, on other hand in selective segmentation our desired object is only detected (Catté, Lions et al. 1992) these both types of segmentation are used in medical field mainly. Our second task is image bias field correction (Song and Zhang 2019), (Ahmed, Yamany et al. 2002), (Bezdek, Hall et al. 1993), (Liao, Lin et al. 2008), (Ahmed 1995) and bias filed is inhomogeneity of an images which frequently occurs in medical images like MRI scans.

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Several models have been proposed in order to perform accurate segmentation but still there are issues. For this purpose we design a novel model for efficiently segmenting and bias field correction. Experimental results on medical, out-door and synthetic images validate the performance of the proposed model in contrast with the existing models. The rest of the paper is organized as: In section 2 we discuss some background work which are related to our work. In section 3 we discuss our proposed model. In section 4 we give experimental evidence of the proposed model. Finally, conclusion is made in section 5.

Background

Here we will discuss some previous techniques used for image segmentation (Ahmed 1995),(Wang, Huang et al. 2010), (Ahmed 1995), (Kass, Witkin et al. 1988), (Zhang, Zhang et al. 2010), (Badshah and Chen 2010), (Ronfard 1994) and bias field correction (Zhan, Zhang et al. 2013), (Shah and Badshah), (Duan, Chang et al. 2015),(Li, Huang et al. 2011), (Chan and Vese 2001). Based on these models we well develop our new model.

Chan-Vese model

For image segmentation, this model uses the level set method (Le Guyader, Apprato et al. 2005) formulation. The CV active contour model energy minimization functional is as follows:

$$F(a_1, a_2, C) = \mu length(C) + \nu_1 \int_{inside(C)} (u - a_1)^2 dx$$
$$+\nu_2 \int_{outside(C)} (u - a_2)^2 dx, \quad (1)$$

where $u(x, y)$ is the given image, C is the unknown boundary, and a_1, a_2 are constants. As the CV model works on images that have twin statistically identical sections and a distinct mean pixel intensity in each. However, there are some drawbacks to the CV model, including as the failure to segment images with intensity inhomogeneity, the comprehension of the position of the initial contour, and we know that intensity inhomogeneities commonly occur in images, i.e. in MR and CT images, therefore for images with inhomogeneity, this model fails to generate efficient results. This model for noise will be improved further. Chan and Vese developed a new model based on linear fitting terms. In order to enhance the CV model inhomogeneity, in the presence of inhomogeneity, Li et al. proposed a model that reveals a considerable obstacle for brain MR image segmentation.

Modified CV model

In this model, the CV-model is somewhat modified by using linear approximation to substitute constants. This is a better model than the CV-model since it works when the intensity changes linearly from one place to another. For images with piecewise constant intensities, this approach also works.

$$F_{linear}^{2D}(a_0, a_1, a_2, b_0, b_1, b_2, \psi) \quad = \mu \int \delta(\psi)|\nabla\psi| dxdy$$

$$+\nu_1 \int_{inside(C)} (u - (a_0 + a_1x + a_2y))^2 dx$$
$$+\nu_2 \int_{outside(C)} (u - (b_0 + b_1x + b_2y))^2 dx, \quad (2)$$

and a_i, b_i can be get by linear system:

$$\frac{\partial}{\partial(a_i)} \parallel u - (a_0 + a_1x + a_2y) \parallel^2 = 0,$$
$$for\ i = 0, 1, 2 \quad (3)$$
$$\frac{\partial}{\partial(b_i)} \parallel u - (b_0 + b_1x + b_2y) \parallel^2 = 0.$$

$$for \ \ i = 0, 1, 2 \qquad (4)$$

This model can works better than the CV model.

Li model

In order to trickle intensity inhomogeneity or bias field in image segmentation, Li et al. (Li, Huang et al. 2011) proposed the following image model:

$$u(x, y) = b.J + n. \qquad (5)$$

For clarity, we shall denote by: $u(x, y) = u(y)$ is observed image. b denote bias field. J represents a bias-free image. Addative noise is represented by the letter n. For the bias field b and the image free form bias field J, Li et al. provide the following assumptions:

• Over the domain of the entire image, the bias field b gradually varies.

• Inside each tissue, the image intensity J is nearly steady. i.e. $x \in \Omega_i$

The Li technique works on images with a high concentration of inhomogeneity, but it fails when inhomogeneity changes suddenly, which is a major flaw in this model.

Modified Li model

For bias field estimation and image segmentation, Badshah and Shah (Shah and Badshah) examine the Li model . (Li, Huang et al. 2011). Consider the Li model, which is written as follows:

$$u(x, y) = b.J + n. \qquad (6)$$

The measure or observed image is $u(x, y) = u(y)$, the bias filed is b, and the desired image free form bias field is J. The fundamental idea behind this model is to identify bias field components from a given image, and then use this model to correct non-uniform intensities efficiently. They hypothesised that the bias field should be smooth and gradually varying over the domain, with constant values in each neighbourhood serving as an approximation. In distinct regions Ω_i, true image J has constant values c_i and $i = 1, \dots, N$. For the expected outcome, they employed the local intensity clustering property. We defined neighbourhood for each point with the property y belonging to Ω.

$$O_y = \{x: |x - y| \leq r\}. \qquad (7)$$

Partition of the entire domain $\Omega = \cup\{\Omega_i\}_{i=1}^{N}$, for $i \neq j$, and a small circular neighborhood $\{O_y \cap \Omega_i\}_{i=1}^{N}$. As the bias field softens and changes with time, so for $x \epsilon O_y$ so we have:

$$b(x) \approx b(y). \qquad (8)$$

We can deduce the following:

$$b(x)J(x) \approx b(x) \times c_i. \qquad (9)$$

The following is the result of the equation stated above:

$$u(x,y) = b(x) \times c_i + n(x). \quad (10)$$

The level set formulation for N=2 is given as:

$$E(b((y), c_1, c_2, \psi(x))) = \int \left(\sum_{i=1}^{2} \int_{\Omega_i} k(yx) \frac{|u(x) - b(y) \times c_i|^2}{(b(y) \times c_i)^2} dy\right) M_i(\psi(x)) d(x). \quad (11)$$

$M_i(\psi(x))$ are member ship functions, with $M_1(\psi(x))$=$H(\psi(x))$ and $M_2(\psi(x))$=$1 - H(\psi(x))$, respectively, where $H(\psi(x))$ is the Heaviside function. When the functional w.r.t ψ, b, c_i $for \ i = 1{,}2$ is minimised, the ideal bias field is:

$$\hat{b} = \frac{\left(\sum_{i=1}^{2} \frac{M_i(\psi(x)) \times u}{c_i^2}\right) * k}{\left(\sum_{i=1}^{2} \frac{M_i(\psi(x))}{c_i}\right) * k}. \quad (12)$$

The optimal value of $\hat{c}_i \ for \ i = 1, 2$ is given as:

$$\hat{c}_i = \frac{\int \left(\frac{1}{b^2} * k\right) u M_i(\psi(x)) dy}{\int \left(\frac{1}{b} * k\right) M_i(\psi(x)) dy}. \quad (13)$$

The minimization w.r.t ψ is given by:

$$\frac{-\partial E}{\partial \psi} = \frac{\partial \psi}{\partial t}. \quad (14)$$

These models have advantages but have some limitations i.e. when bias field is severe then these models fails to produce accurate results.

The Proposed Model

In Li model (Li, Huang et al. 2011) the image with inhomogeneity is given by:

$$u = b.J + n, \quad (15)$$

where u represents the actual image, b represents the component of an image with a bias field, and J represents the desired or true image. n represents noise in the provided image, with n being zero-mean gaussian noise. The following is a hypothesis about the true image:

For $i = 1 \ldots\ldots N$, the actual image roughly takes "N" distinct values, which are sums of constant and linear combinations in N disjoint areas Ω_i. We now describe a strategy for selecting the various regions Ω_i^* and estimating the bias field b^* in the given image. Consider the following:

Energy Formulation

For a given condition the energy functional is then given as:

$$E_{(x)} = \sum_{i=1}^{N} \int_{O_x} |u(y) - b(y)(\lambda_1 c_i + \lambda_2 L_i)|^2 dy, \quad (18)$$

$$E_{(x)} = \sum_{i=1}^{N} \int_{O_x \cap \Omega_i} |u(y) - b(y)(\lambda_1 c_i + \lambda_2 L_i)|^2 dy, \quad (19)$$

$$E_{(x)} = \sum_{i=1}^{N} \int_{O_x \cap \Omega_i} k(x-y)|u(y) - b(x)(\lambda_1 c_i + \lambda_2 L_i)|^2 dy, \quad (20)$$

$k(x-y)$ is the weight kernel function, or k is the truncated Gaussian function used in the Li model, and is denoted by:

$$k(x-y) = \begin{cases} \frac{1}{\alpha} \exp^{\frac{-|x-y|^2}{2\sigma^2}} & for \quad |x-y| \le r \\ 0 & for\ otherwise \end{cases}$$

From above equation we can say that $k(x-y) = 0$ if y doesn't belong to O_x. $E(x)$ gives us the intensities in neighborhoods O_x and in its partition given by $\{O_x \cap \Omega_i\}_{i=1}^{N}$. We propose the following functional:

$$E = \int \left(\sum_{i=1}^{N} \int_{O_x} k(x-y)|u(y) - b(x)(\lambda_1 c_i + \lambda_2 L_i)|^2 dy\right) dx \quad (21)$$

We eliminate Ω_i from the subscript in the specified energy functional because integration is enabled across all domains. Minimizing energy functionals with respect to regions Ω_i, b, c_i, and constants of L_i can be used to segment and estimate the bias field of a given image.

Level set formulation of energy functional

In level set formulation above functional is given by:

$$E = \int \left(\sum_{i=1}^{N} \int k(x-y)|u(y) - b(x)(\lambda_1 c_i + \lambda_2 L_i)|^2 M_i(\psi(x)) dy)\right) dx \quad (22)$$

As E is function of c_i, constant of L_i, b and ψ so E can be written as:

$$E(b, \psi, c_1, c_2, a_0, a_1, a_2, b_0, b_1, b_2) = \int \left(\sum_{i=1}^{N} e_i(x) M_1(\psi(x))\right) dx, \quad (23)$$

here e_i is equal to:

$$e_i = \int k(x-y)|u(y) - b(x)(\lambda_1 c_i + \lambda_2 L_i)|^2 M_i\big(\psi(x)\big) dy \quad (24)$$

Simplification leads to following equation:

$$e_i = u(y) 1_k - 2(\lambda_1 c_i + \lambda_2 L_i) u(y)(b * k) + (\lambda_1 c_i + \lambda_2 L_i)^2 (b^2 * k), \quad (25)$$

here "$*$" shows convolution operation where 1_k is a function given by:

$$1_k = \int k(x-y) dy. \quad (26)$$

Aside from the image space Ω boundary. In the energy of the proposed variation level formulation, the above energy is used as the information term , as evidenced by:

$F(b, \psi, c_1, c_2, a_0, a_1, a_2, b_0, b_1, b_2) =$
$E(b, \psi, c_1, c_2, a_0, a_1, a_2, b_0, b_1, b_2) + \epsilon L(\psi) + \gamma R_p(\psi)$. (27)

The second and third terms are defined as:

$$L(\psi) = \int |\nabla H(\psi(x))| d(x), \qquad (28)$$

and this computes the arc length of Zero level contour of ψ.

$$R_p(\psi) = \int p|\nabla \psi(x)| d(x), \qquad (29)$$

this term is called distance regularization term.

Energy Minimization with respect to ψ, c_i and b

We use the following equation for minimization for proposed functional:

$$\frac{\partial \psi}{\partial t} = -\frac{\partial F}{\partial(\psi)} \qquad (30)$$

The Gateaux derivative of this energy functional is $\frac{\partial F}{\partial(\psi)}$. We may assess this using calculus of variation and describe it as a gradient flow equation:

$$\frac{\partial \psi}{\partial t} = -\delta(\psi)(e_1 - e_2)$$
$$+\upsilon div\left(\frac{\nabla \psi}{|\nabla \psi|} + \gamma div\big(d_p(|\nabla \psi|)\nabla \psi\big)\right) \quad (31)$$

And $d_p = \frac{p^*(s)}{s}$. During the development of level set function as per above condition the constants c_i and bias field b are refreshed by limiting the energy E concerning c_i and b individually which are illustrate under:

We will use the same minimization strategy as in the Li model, which is:

$$c_i^\star = \frac{\int (k*(b))u(x)M_i(\psi(x))d(x)}{\int (k*(b^2))M_i(\psi(x))d(x)}, \ \ here \ \ i = 1,2 \qquad (32)$$

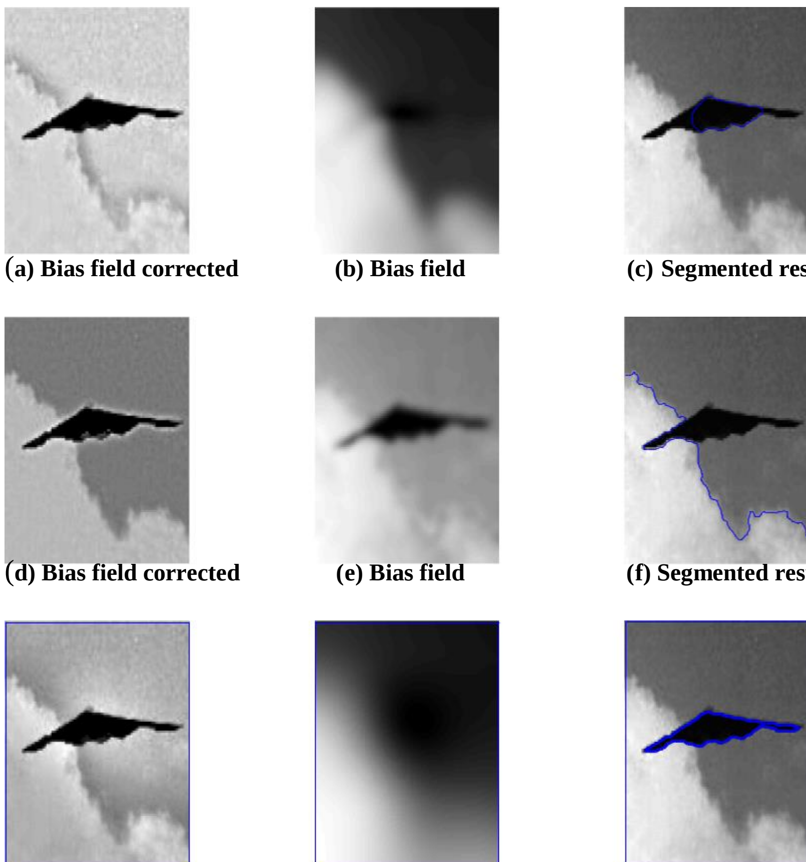
$$b^\star = \frac{k*(\sum_{i=1}^{N}(\lambda_1 c_i + \lambda_2 L_i)M_i(\psi(x))}{k*(\sum_{i=1}^{N}(\lambda_1 c_i + \lambda_2 L_i)^2 M_i(\psi(x))} \qquad (33)$$

$b^\star$ shows estimated bias field and constants of linear equation can be obtained from modified CV model discussed in section 2 and are given by: $\frac{\partial}{\partial(a_i)} \parallel u - (a_0 + a_1 x + a_2 y) \parallel^2 = 0 \ \ and \ \ i = 0,1,2$ (34)

$$\frac{\partial}{\partial(b_i)} \parallel u - (b_0 + b_1 x + b_2 y) \parallel^2 = 0 \ \ and \ \ i = 0,1,2 \qquad (35)$$

Experimental Results

This section discusses some of the new proposed model tests and how they compare to Li et al. (Li, Huang et al. 2011) and Anwar Shah and Noor Badshah model **(Shah and Badshah)** which are briefly addressed in section 2.



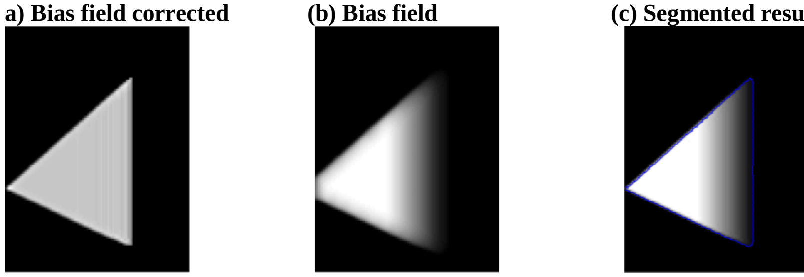
(a) Bias field corrected **(b) Bias field** **(c) Segmented result**

(d) Bias field corrected **(e) Bias field** **(f) Segmented result**

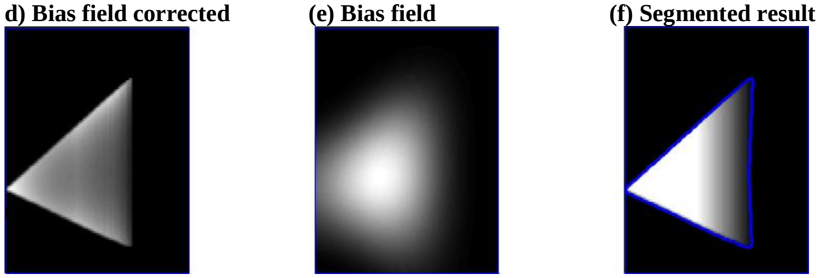
(g) Bias field corrected **(h) Bias field** **(i) Segmented result**

Figure 1: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar (Shah and Badshah) result, and the third row represents our result.

a) Bias field corrected (b) Bias field (c) Segmented result

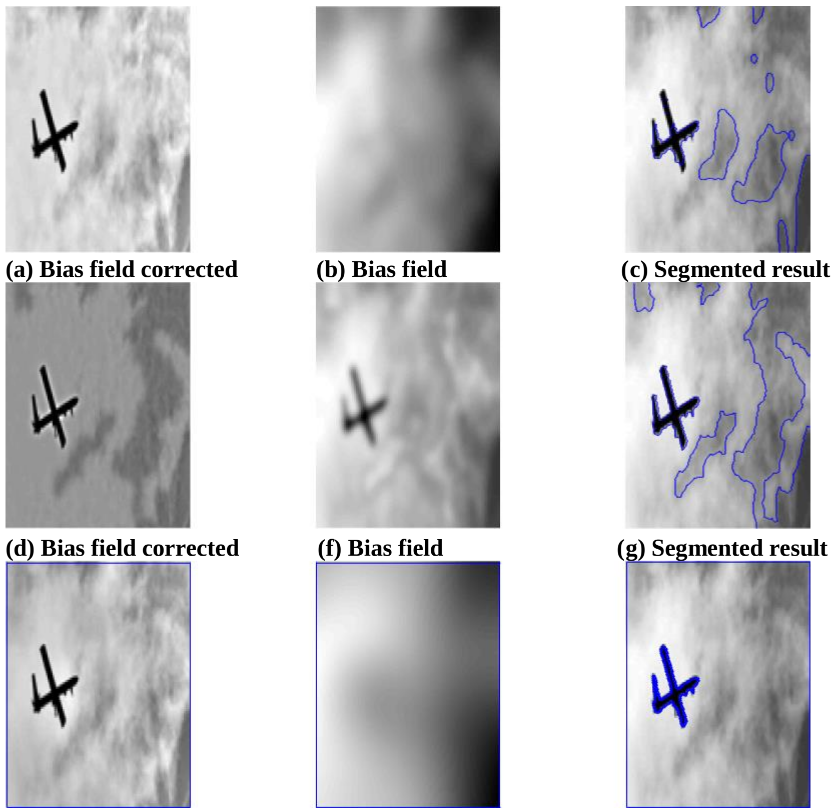


d) Bias field corrected (e) Bias field (f) Segmented result



g) Bias field corrected (h) Bias field (i) Segmented result

Figure 2: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.

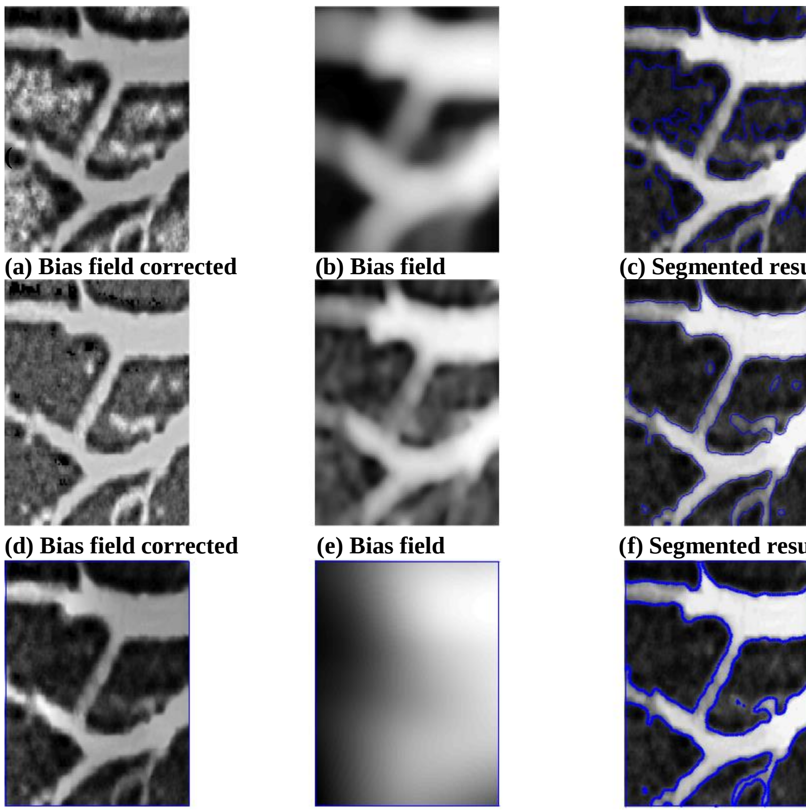


(a) Bias field corrected (b) Bias field (c) Segmented result

(d) Bias field corrected (f) Bias field (g) Segmented result

(g) Bias field corrected (h) Bias field (i) Segmented result

Figure 3: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.

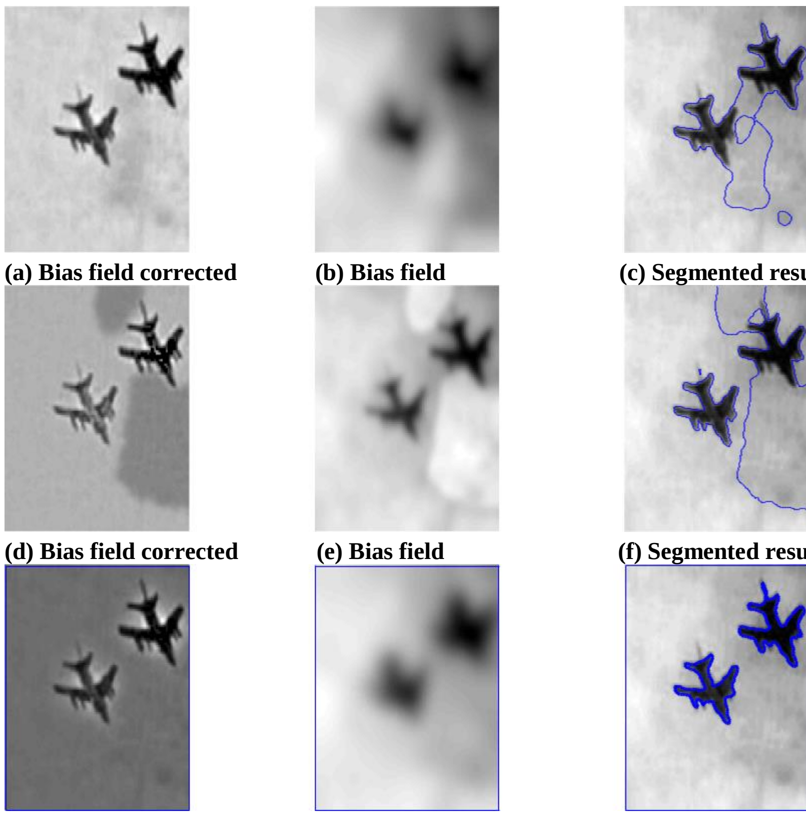


(a) Bias field corrected (b) Bias field (c) Segmented result

(d) Bias field corrected (e) Bias field (f) Segmented result

(g) Bias field corrected (h) Bias field (i) Segmented result

Figure 4: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.

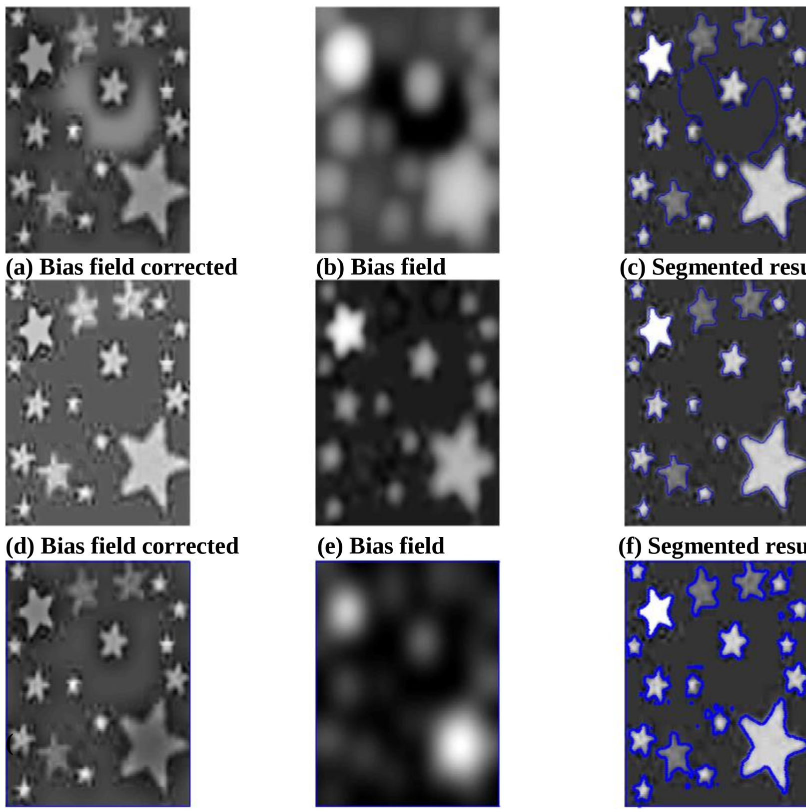


(a) Bias field corrected (b) Bias field (c) Segmented result

(d) Bias field corrected (e) Bias field (f) Segmented result

(g) Bias field corrected (h) Bias field (i) Segmented result

Figure 5: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.

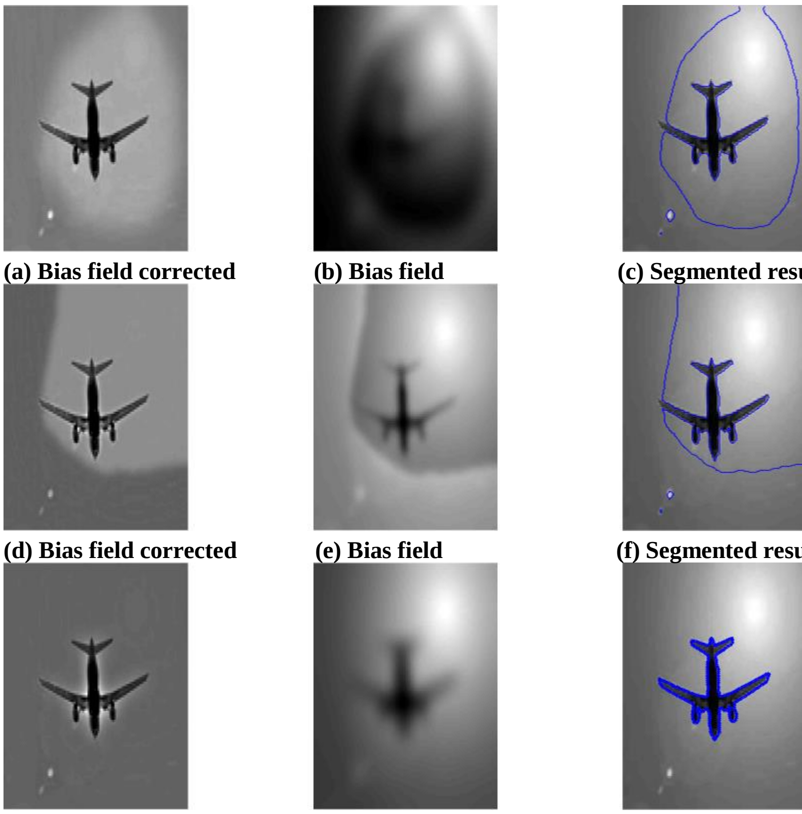


(a) Bias field corrected (b) Bias field (c) Segmented result

(d) Bias field corrected (e) Bias field (f) Segmented result

g) Bias field corrected (h) Bias field (i) Segmented result

Figure 6: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.



(a) Bias field corrected (b) Bias field (c) Segmented result

(d) Bias field corrected (e) Bias field (f) Segmented result

(g) Bias field corrected (h) Bias field (i) Segmented result

Figure 7: The first column represents bias field correction, followed by bias field, and finally segmented result. The first row represents Li (Li, Huang et al. 2011) result, the second row represents Anwar Shah (Shah and Badshah) result, and the third row represents our result.

Conclusions

In this paper we have purposed a novel variational joint image segmentation and bias field correction model for efficiently segmenting and bias field correction. Experimental results on medical, out-door and synthetic images validate the performance of the proposed model. Moreover, in contrast with the state-of-the-art Li model, the proposed model performed far better in medical, out-door and synthetic images.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Data Availability statement
The data that support the findings of the study are available from the corresponding author upon reasonable request.

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