Electron Magnetohydrodynamic (EMHD) Nanofluid flow with triple effects of Chemical reaction, Dufour diffusivity and impermeability of the surface along a Slandering Stretching Sheet

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Abstract

Electro magneto- hydrodynamics (EMHD) has an immense application in machine building and industries. More so, nanofluids have gained notability from several researchers because of their superior thermo- physical properties. This research is based on Magnetohydrodynamics fluid flow over a stretched sheet whose thickness varies which is non-linear. The electric field is also incorporated. Our base fluid is considered to be water, with nanometer-sized Copper (Cu) particles inside as our nanoparticles. We are mainly concerned about the influence caused by some appropriate variables among which are Chemical reaction, induction heat, Dufour diffusivity, Joule heating, variable fluid viscosity, impermeability of the surface, non-uniform heat flux, as well as Viscous dissipation among other parameters on the model. Surfaces with variable thickness have practical significance in our day-to-day real-life applications also in appliance structures, metallurgical engineering and patterns, paper production, atomic reactors, and many more. The research started by formulating some basic governing equations (PDEs) which govern the flow of the fluid, then later converted into a system of non-dimensional ordinary differential equations (ODEs) by the application of appropriate transformations, these equations were solved numerically by R- K4 and Shooting method. The effects of numerous emergent parameters over the fluid flow are portrayed explicitly on graphs as well as some tabulations for validation and computations. Conclusively, it is observed that momentum and temperature are increasing functions with chemical reaction, impermeability, wall thickness, Eckert number, velocity power index, and Biot number parameters respectively. The reverse is the case for temperature and momentum

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with chemical reaction, Dufour diffusivity, and velocity power index respectively. Nusselt number is homogeneously an increasing function.

***Keywords*:** EMHD; nanofluid; Chemical reaction; variable thickness; Dufour diffusivity; impermeability; thermal radiation;

Introduction

Alfven (1942) coined the term Magnetohydrodynamics (MHD) which is a combination of 3 expressions: Magneto which means Magnetic strength while Hydro means water with the 3rd term being dynamics which stands for motion. MHD can simply be referred to as the study of the fluid which electric currents can pass through. The excellent ability of MHD nanofluids to regulate the rate of temperature loss makes them very popular. Over decades, various researchers in the areas of science and technology are greatly concerned over the advantages of MHD nanofluid flow. Sakiadis (1961) is the scientist that discovered fluid flow over a continuous solid surface, he compared it with a sheet of restricted span. He discovered different boundary conditions. He then derived equations for differential momentum after analyzing the flow of the fluid over a constant sheet.

Vajravelu and Cannon (2006) are among several researchers that extended the work of Sakiadis, they explored boundary layer fluids that pass between a non-linear stretchable surface. Lee (1967) used a shrill needle to asses. eventually, he found that the out tinny pointer, slowly reduces the energy and hence displacement, but then become 0 if the thickness returns to 0. Several researchers performed their work on differential fluid nature for varieties of flow configurations of the flow through a stretched sheet. surface with fluctuating thickness is virtually very remarkable in day-to-day implementations and also very useful for paper production, and atomic reactor mechanization. Crane (1070) is also among the researchers that expanded Sakiadis as he looked into the flow of fluids over a stretching surface for momentum which is close to the slit. Where he incorporated internal heat generation/absorption with viscous dissipation effects.

Gupta and Gupta (1977) utilized a stretchable porous and plane sheet of even width and studied flow over the boundary layer. They analyzed distinct occasions of CST (sheet with constant heat) and PST (sheet variable heat). Vajravelu and Rollins (1991) considered and analyzed the heat transmission analyses fluid flow which is laminar as well as non-Newtonian and passes over a stretching sheet in which they used a scaling group of transformations to solve the first order chemical reaction and the variable solute distribution along a stretching surface in the MHD flow of an electrically conducting viscous incompressible fluid.

Choi (1995) considered suspension of nanocrystals (diameters of 1 nm to 100 nm) which colloids. Primarily the base fluids ethanediol, water, and greases. These Nanofluids are endowed for numerous engineering solicitations, such as microchips in computers, a coolant in nuclear reactors, electronic engineering, and hybrid-powered engines, medicinal products, and thermonuclear energy besides its frequent use in the arena of contemporary nanotech., solar radiation, heat exchanger tubes, with many more. Cortell (2007) considered temperature exchange transfer diffusion for adhesive nanofluid which passes over a non-linear stretchable surface, he used the R-K and shooting methods. The comportment of MHD flow nearby an inertia point over a stretching surface was studied by Ishak et al. (2008) who considered a nanofluid rolling on a stretched sheet. Khan and Pop (2010) are the 1st to utilize Buongiorno's model in anatomizing nanofluid flow.

A further imperative element that influences the temperature change physiognomies of the fluid flow is the impermeability of the flowing surface. The concept of heat transfer with impermeable surfaces takes widespread solicitations in an electronic device. Recently, scientists and engineers are passionately absorbed in an escalation of the productivity of numerous automatic structures and engineering technology. Fang et al. (2012) discovered fluid flow over an impermeable sheet over a stretched surface for non-uniform thickness. Nadeem and Lee (2012) discussed temperature exchange analysis by modeling the boundary layer flow through a porous stretched surface which is influenced by thermophoresis as well as Brownian diffusion, then applied HAM to find its solution. Khader and Megahed (2013) scrutinized two-dimensional Newtonian fluid flow. Ibrahim and Shankar (2013) studied Magneto hydrodynamics fluid flow with slip conditions by the use of Buongiorno's method. Malvandi et al. (2014) examined inertia points through stretchable surfaces along the influence of glide pace. Khan et al. (2015) utilized an implicit finite difference method to discover the impacts of thermophoresis and Brownian motion on a three-dimensional nanofluid flow over an exponentially stretchable surface.

Joule heating can be referred to as heat formed as a result of the flow of the current of electricity which passes a conductor. It is used differently in everyday applications, which include the radiant string of a glowing bright tuber, fuses, hotplates, electrical tabletop, and many more. Hsiao (2016) scrutinizes the influence of a Brownian motion, heat generation/ absorption, viscous dissipation, thermophoresis, and radiation in EMHD together in a 2D incompressible fluid. Daniel et al.

(2018) deliberated on the impact of numerous parameters on EMHD nanofluid flow. Elbashbeshy et al. (2018) studied the behavior of Maxwell fluid on a slandering stretched surface. Ali et al. (2019) inspected the connection concerning nanofluids and microwaves. Bognáret al. (2020) used the 4th – 5th R–K method to study pseudo-plastic nanofluids laterally a permeable sheet. Adem (2020) studied EMHD boundary layer flow on a stretching surface. Ali et al. (2021) explored the linkage between the grouping and thermal conductivity phenomena in nanofluids. Alazwariet al. (2021) analyzed temperature exchange over a mountingly stretchable sheet over a three-dimensional. Colangelo al. (2021) evaluated 2nd-grade nanofluids flow.

In recent years, Ali et al. (2022) studied the influence of Jeffery MHD fluid flow through the stretched sheet that has fluctuating widths. Ali and Ghori (2022) surveyed the tendering fluence in Cattaneo–Christov and radiation effects over Magneto hydro dynamics nanofluid flow, which includes microbes. Souayeh et al. (2022) conversed the mixed nanofluids via a peristaltic station in a stimulus of Electron magneto hydro dynamics, gyrotactic microbes, radiation, and motivation energy. about the relationships between a laminar flow surface with a porous surface. Gaponov (2009). In another development, Chatterjee et al. (2022) reviewed some current revisions on the fomite spread of COVID-19, caused by the innovative coronavirus. Gupta and Sharma (2017) in their paper investigated the stable Two-Doff MHD fluid flow.

Safya et al. (2021) established a mathematical model to study the effects of heat source on MHD, he applied three-dimensional chemical reaction pair tension nanofluid flow caused by the stretched sheet. The fluid flow in this research is under the control of Brownian motion and Thermophoresis force on the energy and concentration profiles. Barakeh et al. (2022) examined the impact of Hall current and Soret number on over EMHD nanofluids. Venkata et al. (2022) highlighted the influence of MHD Casson fluid flow over a stretching sheet. Ali et al. (2022) studied the growing effects of Joule heating, induction heat, variable heat flux, etc. with the predisposed applications of non-linear stretched surface and thickness which varies in electromagnet hydrodynamics. In a related development, Madaki et al. (2017, 2019, 2021 and 2022), Hussaini et al. (2022, 2022, 2022, and 2021). Moreover, Madaki and Hussaini (2021) had extensively discussed the phenomenon in relation to the nanofluids.

In this paper, our goal is to determine after considering the above-mentioned pieces of literature and extensively studying the industrial applications, the influence caused by some appropriate quantities during the heat transfer process. Observations show that linear stretchable surface has substantially been used in the analysis of heat transmission. however, as truthfully as possible, we presumed this study was an initial effort that considered simultaneous influences of chemical reaction, Dufour diffusivity, variable viscosity, viscous dissipation, and impermeability of the sheet. This brought about the motivation behind the present study which is mainly about the influence of the aforementioned parameters over the electro-magneto hydrodynamics (EMHD) overflow of nanofluid on a stretchable sheet. To this end we started by investigating the substantial characteristics of the model, we then inspected the model and traced all the somatic characteristics of the model. The model is made up of basic laws and governing equations, which are non-linear partial differential equations (PDEs). Which are then converted into a dimensionless form of non-linear Ordinary differential equations (ODEs) by the use of similarity transformations and non-dimensional variables. We utilized the R- K4 technique along with the shooting method to obtain the numerical elucidation of the resulting non-dimensional system of equations. We utilized Graphs to visualize the impact of relevant parameters in the model.

Problem Formulation

We considered a two-dimensional magneto hydrodynamic (MHD) flow of a nanofluid over a non-linear stretchable surface and varying width considering electric field. $U_w(x) = b\,(x + c)^{n}$ is the stretched velocity of the sheet. In this case b, c, n stands for constant. We examined at wo-dimensional system for which x-axis which is analogous while y-axis is placed normal to the stretched sheet. The surface was assumed not to be flat; hence, it has a variable thickness given by $y = N(x+c)^{\frac{1-n}{2}}$,where N denotes minor factor which guarantees the surfaces maintained reedy enough to circumvent an obvious density difference along its distance. 2equalsbut opposite forces cause the enlargement of the surface from a slit, the flow is in the accordance with the direction of electric field and Magnetic field. By Ohm's law: σ (E + V × B) =J, such that (J- denotes current density), (σ the electrical conductivity) and (V stands for velocity of the fluid). $B(x)=B_0\,(x+c)^{\frac{n-1}{2}}$ is the magnetic intensity, $E(x)=E_0\,(x+c)^{\frac{n-1}{2}}$ is the electric field, $T_w = T_\infty + T_0(x+c)^{\frac{1-n}{2}}$ denotes the temperature of the surface. Taylor series expansion will be employed to convert (T^4) to be our ambient temperature (T_∞). The physical model of the fluid flow is discussed below, in fig. 1.

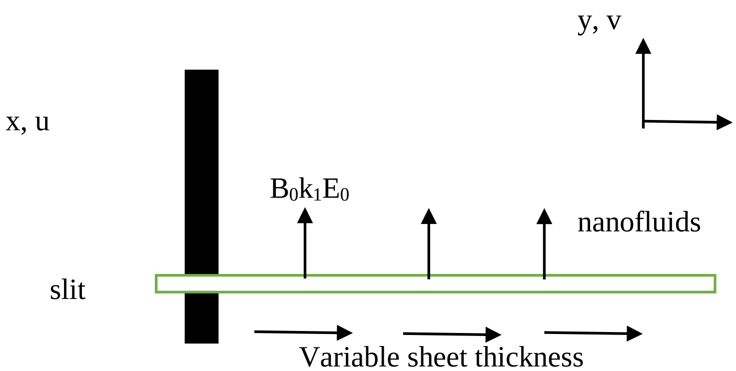

Figure 1: Schematic configuration of the model

This work is based on Ali et al. (2022), and according to their findings, the model is made up of continuity, momentum, energy and nanoparticle concentration equations after the modifications,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+\sigma_{nf}\left(E(x)B(x)-B(x)^2u\right)+\frac{\mu_e}{k}u, \tag{2}$$

$$(\rho Cp)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2+q'''$$
$$+\sigma_{nf}\left(uB(x)-E(x)\right)^2+D_{TC}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=k(C-C_\infty), \tag{4}$$

And these are the boundary conditions:

$$v=0, u=U_w(x)=b(x+c)^n, C=C_w, -k\frac{\partial T}{\partial y}=h(T_w-T),\ at\ \ y=N(x+c)^{\frac{n-1}{2}}$$
$$u\to 0, T\to T_\infty, C\to C_\infty\ \ as\ \ y\to\infty, \tag{5}$$

In this case, u and v stands for the velocity components, C, $T, \sigma_{nf}, \alpha_{nf}=\dfrac{k_{nf}}{(\rho Cp)_{nf}}$ are the concentration, temperature, electrical conductivity, and thermal diffusivity of the nanofluid. While, ρ_{nf}, is the density, σ^* and k^* are Stefan- Boltzmann constants with mean absorption constant, accordingly. The thickness coefficient of the fluid can be defined as follows (2022)

$$\mu(T) = \mu^*[a_1 + b_1(1-\theta)(T_w - T_\infty)], \tag{6}$$

Here, a_1, b_1, are constants, while μ^* refers the reference viscosity. q''' as it appears in eq.3 stands for the non- uniform heat flux which is as follow:

$$q''' = \frac{k_{nf} U_w(x)}{v_f (x+c)} \left[A^*(T_w - T_\infty) + B^*(T_w - T_\infty) \right], \tag{7}$$

In this case, A^* is the space dependent coefficient, B^* stands for the time heat flux parameter. Consequently, by applying the similarity transformations, eq.1 is satisfied automatically, and eqs.2- 5 becomes These are the non- dimensional quantities

$$v = -\sqrt{\frac{(n+1)vb(x+c)^{n-1}}{2}} \left(f(\xi) + \xi f' \xi \left(\frac{n-1}{n+1} \right) \right), u = b(x+c)^n f'(\xi),$$

$$\xi = \sqrt{\frac{(n+1)b(x+c)^{n-1}}{2v}}\, y, T = T_\infty + \left(T_w(x) - T_\infty \right) \Theta(\xi) \tag{8}$$

$$A_0\left[\left(a_1 + A(1-\theta) \right) f''' - A\theta' f'' \right] + A_1\left[f f'' - \frac{2n}{n+1}(f')^2 \right] + A_2 M(E_1 - f') - k_1 f' = 0 \tag{9}$$

$$\left(A_4 + \frac{4}{3}Rd \right)\theta'' + \Pr\left[A_0 Ec(f'')^2 + A_2 EcM(f' - E_1)^2 + A_3 f\theta' - A_3 f'\theta\left(\frac{1-n}{1+n} \right) \right]$$
$$+ A_4 \frac{2}{1+n}\left(A^* f' + B^*\theta \right) + Nd\varphi'' = 0, \tag{10}$$

$$\varphi'' + \lambda\varphi = 0, \tag{11}$$

These the boundary conditions:

$$f(\eta) = \left(\frac{1-n}{1+n} \right)\alpha,\ f'(\eta) = 1, \theta'(\eta) = -\delta(1-\theta(0)), \varphi(0) = 1 \ at \ \eta = 0$$
$$f'(\eta) = 0, \theta(\eta) = 0, \varphi(\infty) = 0 \ as \ \eta \to \infty, \tag{12}$$

These are the non- dimensional parameters which are involved in eqs. (9)- (12), the magnetic parameter M, electric field parameter E_1, Prandtl number Pr, Dufour diffusivity Nd, wall thickness parameter α, dimensionless variable viscosity A, δ is the Biot number, radiation parameter Rd, Eckert number Ec, chemical reaction λ and impermeability parameter k_1, they are defined as follows:

$$M = \frac{2\sigma_f B_0^2}{\rho_f b(n+1)}, A = b_1(T_w - T_\infty), E_1 = \frac{E_0}{B_0 b(n+1)^n}, \Pr = \frac{v_f(\rho c_p)_f}{k_f}, \alpha = N\sqrt{\frac{b(n+1)}{2v_f}},$$
$$\delta = \frac{h}{k_f}\sqrt{\frac{2v_f}{(n+1)^b}(x+c)^{1-n}}, Rd = \frac{4\sigma^* T_\infty^3}{k^* k_f}, Ec = \frac{b^2(x+c)^{2n}}{(c_p)_f(T_w - T_\infty)}, Nd = \frac{D_{TC}(C_w - C_\infty)}{\alpha(T_w - T_\infty)},$$
$$\lambda = \frac{k_1 \rho_f (C_w - C_\infty)}{bC_w}, k_1 = \frac{abx\mu_e}{k} \quad (13)$$

Physical Quantities

This is the mathematical formulations:

$$Nu_x = \frac{(x+c)q_w}{k_f(T_w - T_\infty)} \text{ where } q_w = -k_{nf}\left.\frac{\partial T}{\partial y}\right|_{y=N(x+c)^{\frac{n-1}{2}}},$$
$$C_f = \frac{\tau_w}{\rho_{nf}U_w^2} \text{ where } \tau_w = \mu_{nf}\left.\left(\frac{\partial u}{\partial y}\right)\right|_{y=N(x+c)^{\frac{n-1\backslash\backslash}{2}}} \quad (14)$$

The dimensionless Nusselt number as well as the dimensionless skin friction coefficient was obtained by substituting eq. (8) into eq. 14 as follows:

$$\mathrm{Re}_x^{-1/2} Nu_x = -A_4\sqrt{\frac{n+1}{2}}\theta'(0), \mathrm{Re}_x^{1/2} C_f = \frac{A_0}{A_1}\sqrt{\frac{n+1}{2}} f''(0), \quad (15)$$

Where the dimensionless local Reynold number Re_x, $\mathrm{Re}_x = U_w(x+c)/v_f$

Thermo- physical properties

Base fluid's thermo-physical properties are given in table1, whereas experimental relationship between thermo- physical properties of the fluid is given in table2:

Table 1

Base fluid and the nanoparticle thermophysical numerical values

S/no	Base fluid (H_2O)	Copper (Cu)	Physical properties
1.	997.1	8933	$\rho(kg/m^3)$
2.	4179	385	$c_p(J/kg.K)$
3.	0.613	401	$k(W/mk)$
4.	0.05	5.96×10^7	$\sigma(/\Omega m)$

From Table2 k_p, is the particle's thermal conductivity k_f, ρ_p, ρ_f, thermal conductivity of the base fluid, density of the nanoparticle and density of the base fluid respectively. ρ_{nf} density of the nanoparticle ,$(\rho c_p)_{nf}$ is the specific heat capacity, μ_{nf} is denoting the dynamic viscosity, k_{nf} is the thermal conductivity of the nanofluid,

$\phi, (\rho c_p)_p, (\rho c_p)_f$ are the volume fraction of nanoparticles, the heat capacities of the nanoparticles and base fluid.

Table 2
Nanoparticle thermophysical properties

S/no	nanofluid	Properties
1.	$\sigma_{nf} = 1 + \dfrac{3\phi\left(\dfrac{\sigma_p}{\sigma_f} - 1\right)}{\left(\dfrac{\sigma_p}{\sigma_f} + 2\right) - \left(\dfrac{\sigma_p}{\sigma_f} - 1\right)\phi}\sigma_f$	Electrical conductivity
2.	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p$	Density
3.	$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_p$	Heat capacity
4.	$\mu_{nf} = \dfrac{\mu_f}{(1-\phi)^{2.5}}$	Viscosity
5.	$k_{nf} = \left(\dfrac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)}\right)k_f$	Thermal conductivity

Results and Discussion

Equations 9 to 11 are solved numerically subject to the boundary conditions Equation (12) by the use of Maple. The correctness and sturdiness of this result is first ensured. Furthermore, as are check to validate our numerical results, the obtained values are compared to published results as will be explained in table 3. These results pertain to a circumstance when the stretching sheet is at a constant temperature and the Brownian and the thermophoresis effects are absent, that is, the fluid is a regular fluid with no nanoparticles.

In table3 we compare the present result with the published work of Ali et al. [36] for the values of $-f'(0)$, when $\alpha = 0.25$, $E_1 = 0$, $\varphi = 0$, $A = 0$, $M = 0$ and $a_1 = 1$ with numerous values of the velocity exponential (n). Which is clearly visible there is a fascinating agreement between the published results with the present result.

Graphical results: Momentum profile

This segment will provide the discussion on the influence and behavior of some important parameters over the profiles of momentum, energy, nanoparticle concentration as well as Nusselt number and Sherwood number for different values of the influencing parameters. These parameters include: velocity power index (n), radiation parameter

(Rd), chemical reaction parameter (λ), viscous dissipation (A), parameter for the wall thickness (α), impermeability of the surface (k_1), Eckert number (Ec), Dufour diffusivity parameter (Nd) and heat transfer Biot number (δ) etc.

Table 3
Comparison of results for non- dimensional momentum with various values of velocity exponential factor

n	Ali et al. (2022)	Present results
10	1.143220	1.143321
9	1.140575	1.140392
7	1.132588	1.137947
5	1.118587	1.118587
3	1.090490	1.090490
1	1.000001	1.000001
0.5	0.933828	0.933881
0	0.784284	0.784279
-1/3	0.500000	0.500000
-0.5	0.083289	0.083289
-0.51	0.038484	0.038484
-0.55	-0.197647	-0.197647
-0.60	-0.850207	-0.850207
-0.61	-1.224426	-1.224426

Table 4
Computing the reduced Nusselt number $-\left(\theta'(0)\right)$ *when n = -0.6, λ =0.5, and δ = 3.*

Pr	Nur Nd = 0.1	Nur Nd = 0.2	Nur Nd = 0.3	Nur Nd = 0.4	Nur Nd = 0.5
0	-0.552200	-1.007627	-1.463054	-1.918481	-2.373908
0.2	-0.679019	-0.913824	-1.148629	-1.383433	-1.618238
0.3	-0.737089	-0.920091	-1.103092	-1.286094	-1.469096
0.5	-0.842231	-0.979180	-1.116130	-1.253079	-1.390029
0.7	-0.929169	-1.051506	-1.173843	-1.296180	-1.418517

Table 5
Computing the reduced Sherwood number $-\left(\varphi'(0)\right)$ *when Pr =0.7, n = 0.5, α = 0.25*

Ec	Shr λ = 0.1	Shr λ = 0.2	Shr λ = 0.3	Shr λ = 0.4	Shr λ = 0.5
0.1	-15.285596	-0.109560	0.525649	-15.279054	-0.703602
0.2	-15.285596	-0.109560	0.525649	-15.279054	-0.703602
0.3	-15.285596	-0.109560	0.525649	-15.279054	-0.703602
0.4	-15.285596	-0.109560	0.525649	-15.279054	-0.703602
0.5	-15.285596	-0.109560	0.525649	-15.279054	-0.703602

Table 6

Computing the reduced Nusselt number $\left(-\theta'(0)\right)$,

Sherwood number $\left(-\varphi'(0)\right)$, *when Pr =0, δ = 3, Ec=0.5, Rd = 0.25, M=5*

λ	**Nd**	$\left(-\theta'(0)\right)$	$\left(-\varphi'(0)\right)$
0.1	1	-3.245093	0.009814
0.2	1	-3.770232	-0.988086
0.3	1	-2.410968	0.207326
0.4	1	-10.716613	-10.178765
0.5	1	-2.714169	-0.289810
0.6	1	-2.179750	0.556948
0.6	2	-0.434139	0.556948
0.6	3	1.311471	0.556948
0.6	4	3.057082	0.556948
0.6	5	4.802693	0.556948
0.6	6	6.548304	0.556948

The influence of velocity power index is depicted on Fig.2 in which an increase in parameter brought about a slight slows down in the momentum of the fluid. A hike in (λ) enhances stretching momentum, which brought about deformation of more fluids, in this case, for that purpose, the velocity boundary layer is elevated as seen on fig.3. The influence of impermeability parameter is depicted on fig. 4, A large impermeability parameter enhances the velocity profile. The presence of k_1 allows the exchange of fluid particles among regions within the boundary layer. Now, increasing the values for k_1 expands the pore size, hence providing space for more movement of fluid particles. Naturally, the wall thickness is placed within the range of 0.5mm to 4mm. A rudimentary scheme standard is to retain wall thicknesses as tinny as probable. The impact of well thickness parameter on the momentum profile is depicted on fig. 5, it is clearly displayed that an increase in the parameter enhances the flow of the fluid.

Temperature profile

The wall thickness will be in range of 0.5mm to 4mm typically. In specific cases, wall thicknesses that are either smaller or bigger also occurs. A basic design guideline is to keep wall thicknesses as thin and as uniform as possible. Where varying wall thicknesses are unavoidable for reasons of design, there should be a gradual transition. The influence of wall thickness parameter on the profile of temperature is depicted on fig. 6, in this case the fluid flow features approach the surface and they drastically condensed, which produces reduction for the thickness of the wall in respect with the momentum in every location in the zone, and hence, increase in the wall thickness parameter enhances the temperature. The effects of Ec energy profile are displayed on fig. 7, it is clearly visible that an increase in the values of Eckert number result to rise in the thickness of the thermal boundary layer and hence the temperature of the system is enhanced. This is because, the energy variance among the fluid surface and the neighboring air decreases. Fig. 8 is about the effect of chemical reaction over the temperature profile, it can be seen there that an increase in the chemical reaction parameter produces a decrease in the temperature profile.

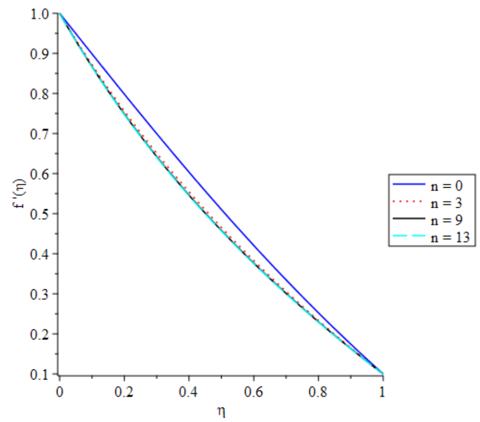

Figure 2. effects of velocity exponential factor (n) on the momentum profile.

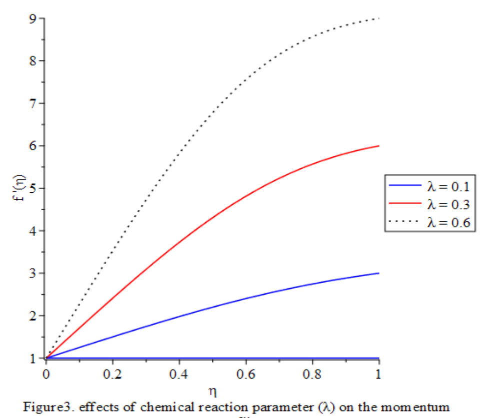

Figure3. effects of chemical reaction parameter (λ) on the momentum profile.

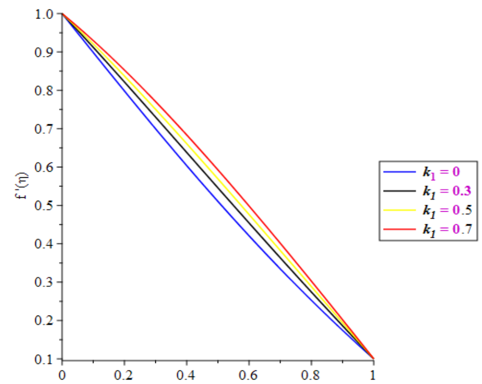

Figure4. effects of impermeability parameter (k1) on the momentum profile.

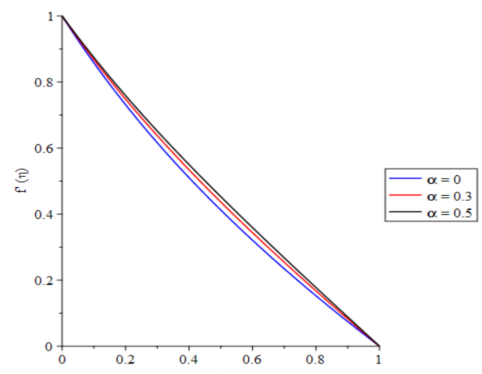

Figure5. effects of Wall thickness parameter (α) on momentum profile

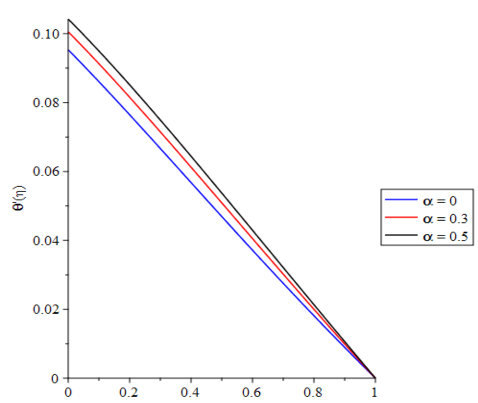

Figure6. effects of Wall thickness parameter (α) on temperature profile.

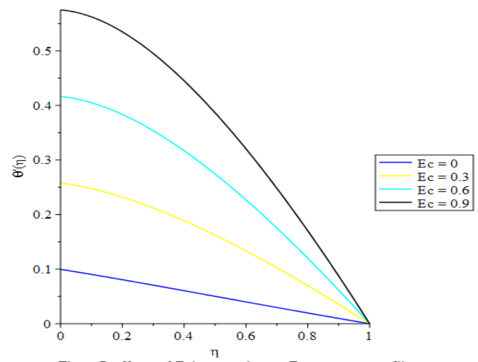

Figure7. effects of Eckert number on Temperature profile.

The variations ascertain the comportment of the smear sheet. The influence of n on the energy profile is portrayed on fig. 9. For an increase in the values of the velocity power index enhances the temperature of the system. An increase in the Dufour diffusivity parameter decreases the temperature, this is depicted on fig. 10. The effect of Biot number parameter on the profile of temperature is depicted on fig. 11, in this case, an increase in the Biot number parameter, rises the temperature of the system.

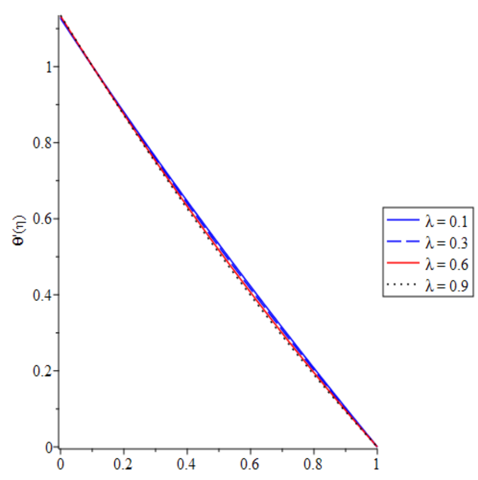

Figure8. effects of Chemical reaction on Temperature profile.

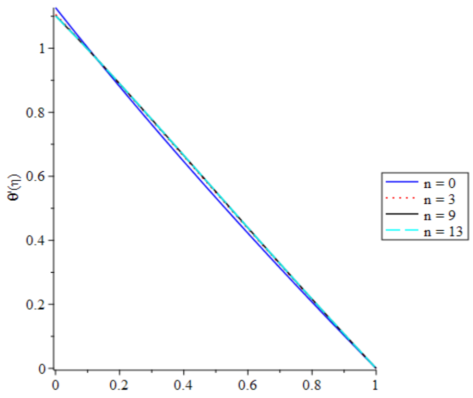

Figure 9. effects of velocity exponential factor (n) on the Temperature profile.

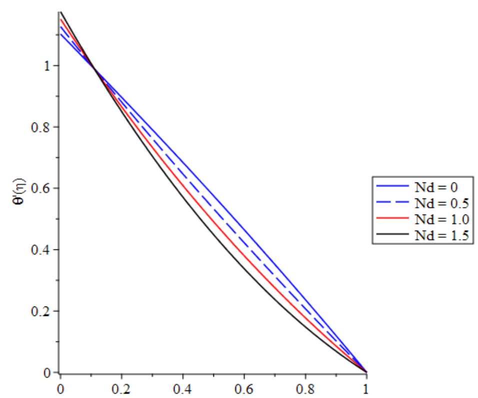

Figure10. effects of Dufour diffusivity parameter (Nd) on the Temperature profile.

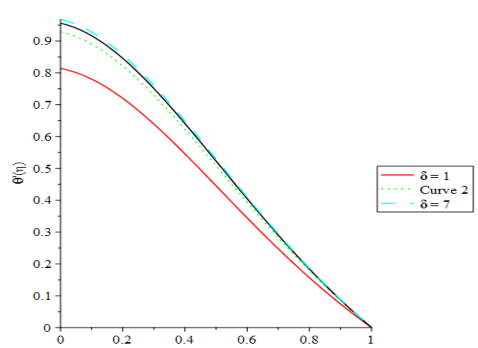

Figure11. effects of Biot number parameter (δ) on temperature profile

Nusselt Number Profile

The influence of space- dependent co-efficient is depicted on fig.12. in this case, it is clearly displayed that for any increase in the values of the parameter it produces an increase in the values of Nusselt number. On fig.13 which is on the effects of velocity power index, such that for any increment in the velocity power index rises the values of the Nusselt number.

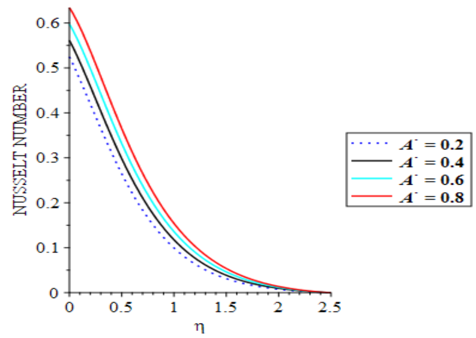

Figure12. effects of Space- dependent co-efficient parameter (A*) on Nusselt number profile

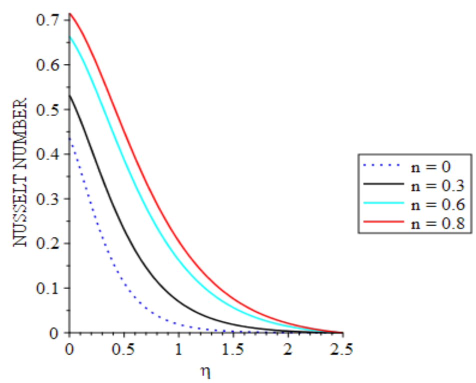

Figure13. effects of velocity power index parameter (n) on Nusselt number profile

Concentration (Volume fraction) profile

The influence of chemical reaction parameter on the nanoparticle concentration profile is depicted on fig. 12, in this case, for any rise in the values of chemical reaction, produces a rise in the concentration of the nanoparticle.

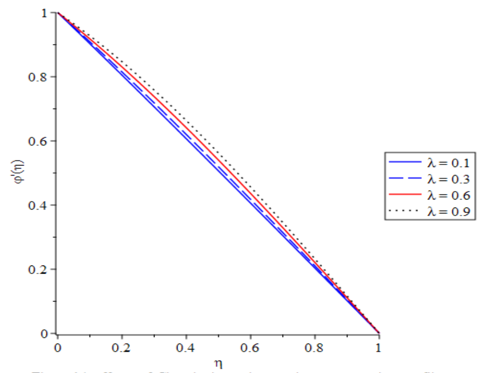

Figure14. effects of Chemical reaction on the concentration profile.

Conclusion

The research considered a 2-D Electro magneto- hydrodynamics fluid flow over a stretched sheet with unsteady thicknesses. The system of PDEs is transformed to ODEs by applying non- dimensional parameters and similarity transformations. We numerically solved the obtained ODEs via Runge- Kutta- Fehlberg. By visualizing momentum, energy and nanoparticle concentration on the influence of some important parameters were studied and analyzed. Below are some of the observations from this research:

- Both momentum and temperature profiles are enhanced with an increase in the values of wall thickness parameter.
- Both temperature and nanoparticle concentration increase with an increase in the values of chemical reaction.
- The temperature is increased with an increase velocity power index while reverse is observed for momentum.
- A hike in the values of Bi rises the temperature.
- A hike in the impermeability parameter speeds up the fluid flow.
- The temp. is enhanced with a hike in the Dufour diff. parameter.
- Values of the Nusselt number increases with an increase in both space- dependent co-efficient as well as velocity power index.

Authors Contribution

A. G. Madaki and S.K. Alaramma:

Conceptualization, methodology, supervision, formal analysis, investigation,

A. A. Hussaini, A.M. Musa and A.A. Tata:

Software, writing- unedited and edited version, validation, resources.

The authors have unanimously read and agreed to the draft and then final edition of the typescript.

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Conflict of Interest

This is to proclaim that the authors don't have any form of conflict of interest

Nomenclature

A Wall thickness parameter
a Variable viscosity parameter
A^* Space-dependent co-efficient
B^* Time-dependent co-efficient
B_o Magnetic parameter
c_p Heat capacity
Ec Eckert number
E_0 Electric field parameter
M non dimensional Magnetic field parameter
E_1 Electric field parameter
T Temperature of the Fluid
T_w Surface temperature
T_∞ Ambient temperature
Rd Thermal Radiation parameter
Pr Prandtl number
δ Biot number
q_r Radiative heat flux
k^* Mean absorption coefficient
n Velocity power index parameter
u, v Components of velocity
N Co-efficient related to stretching sheet

Greek symbol

θ Dimensionless temperature (K)
ρ Density
v Kinematic viscosity
μ Dynamic viscosity
σ Electric conductivity
σ^* Stefan–Boltzmann constant
k Thermal conductivity
η non-dimensional similarity variable
U_w Stretching velocity

Subscript

n_f Nanofluid
f Base fluid

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