An Attempt to Revamp Vogel's Approximate Method for Optimality of Transportation Problems

Fozia Shaikh*, Afaque Ahmed Bhutto†, Muhammad Memon‡,
Syed Feroz Shah§, Khadija Shaikh**

Abstract

The transportation of products from manufacturers to destinations, while ensuring profitability, constitutes a pivotal facet of industrial operations. Nevertheless, the inherent unpredictability of transportation costs poses a challenge. Therefore, a linear programming is devised to mitigate overarching expenses. Solution of a transportation problem requires the initial feasible solution to get an optimal solution. Vogel's Approximation Method, which is very effectual in the literature, however difficulty arises in the algorithm when two smallest costs have same magnitude. In that instance, this study aims to revamp Vogel's method for balanced problems by employing a statistical approach to prevail over the difficulties of the Vogel Approximation Method (VAM) and to identify a feasible solution that is closer to the optimal solution than VAM. A range of literature examples have been investigated to validate the legitimacy and efficacy of the suggested algorithm. Additionally, the findings indicate significant achievements in cost reduction. The Initial Basic Feasible Solution of the revamped approach has been proven to be perfectly optimal, nearly optimal, equivalent to VAM, or superior to conventional methods.

Keywords: Least Cost Method; North-West Corner Method; Vogel Approximation Method; Balanced Transportation; Initial Basic Feasible Solutions.

Introduction

Linear programming involves optimizing linear functions under specific conditions, aiming to maximize or minimize values. This optimization technique is commonly utilized in solving transportation problems within the realm of operations research. These transportation issues include both balanced and unbalanced scenarios, seeking initial basic feasible solutions (IBFS) and cost minimization, ultimately aiming for optimal outcomes. Operation research models in transportation

---

*Corresponding Author: Department of Basic Sciences & Related Studies, Mehran University of Engineering & Technology, Jamshoro 76062, Pakistan, fozia.shaikh@faculty.muet.edu.pk

†Department of Basic Sciences and Related Studies, The University of Larkano, Larkana 77150, Pakistan, afaq_bhutto@uolrk.edu.pk

‡Department of Basic Science & Related Studies, Quaid-e-Awam University of Engineering, Science, & Technology, Nawabshah 67450, Pakistan, muhammadmemon@quest.edu.pk

§Department of Basic Sciences & Related Studies, Mehran University of Engineering & Technology, Jamshoro,76062, Pakistan, feroz.shah@faculty.muet.edu.pk

**Department of Basic Sciences & Related Studies, Mehran University of Engineering & Technology, Jamshoro,76062, Pakistan, khadijaDOqadir@gmail.com

problems focus on cost optimization to efficiently reach destinations. Balanced transportation problems occur when supply matches demand, while unbalanced scenarios arise when supply and demand differ. Hitchcock (1941) initially developed transportation problem algorithms during World War II, and these problems are crucial in supply chain management, optimization techniques, and various fields like Mathematics, Engineering, Business, and Economics.

There's a plethora of optimization techniques utilized for both balanced and unbalanced transportation problems, aiming to obtain IBFS and ultimately reaching optimal or near-optimal solutions. Some methods yield optimal solutions, while others focus on providing basic feasible solutions. Examples include the Vogel Approximation Method (VAM), Least Cost Method (LCM), North-West Corner Method (NWCM), Matrix Minima Method, Invariant VAM (IVAM), and Modified VAM (MVAM) for obtaining IBFS (Eiselt & Sandblom, 2022). For optimal or near-optimal solutions, techniques such as Modi Method, Zero Point Method, and Harmonic Mean Technique Method are employed. The scope of this study is limited to the previous research on IBFS techniques since it involves suggesting an effective IBFS method or approaches for Transportation Problems (TPs). IBFS techniques have been introduced by several researchers (Kumar et al., 2018; Hosseini, 2017; Ullah et al., 2016; Hossain & Ahmed, 2020). Studies have been proposed on traditional methods to minimize the transit cost for both homogenous and non-homogenous items. Crankson et al. (2023) modified the Vogel's method to find feasible solution of mining company Tarkwa. Research has been conducted on the issue of drug delivery from drug manufacturers to various warehouses to minimize delivery times and costs while meeting destination requirements (Mallick et al., 2023).

Among all the IBFS techniques included in an operations research textbook, the VAM has gained prominence in a number of practical uses as it offers very effective IBFS or an almost ideal solution. The lowest cost is not always guaranteed by the greatest penalty notion in the basic form of the VAM for obtaining IBFS, though, as the difference between the first two pairs of lowest costs can equalize if one of the pairs is lower than the other. This has led to the proposal of a novel IBFS approach by several researchers (Juman & Hoque, 2015; Amaliah et al., 2019; Karagul & Sahin, 2020; Babu et al., 2020), who have compared it to VAM in terms of performance efficiency. Korukoglu & Balli (2011) propose a MVAM. Rather than choosing the greatest penalty cost row/column, MVAM selects the three largest penalty cost rows or columns. The MVAM technique generates three allocations based on the three rows or columns with the highest penalty costs; of these, one is approved in relation to the least transportation cost of the three allocations. Jain (2015) introduced the Maximum Difference Approach as an innovative method to identify IBFS in TPs, offering a direct and effective solution that generally surpasses Variance Analysis MVAM. Amaliah et al. (2022) proposed a study on the transportation cost and introduced new approach Supply Selection Method to reached to the optimality of the balanced problems.

In the transit problems, each source has several destinations to which items must be sent while taking capacity constraints into account. The fundamentally feasible solution must be satisfied by the sources' capacity and the destinations' requests. The goal is to ascertain the amount that, to save overall transportation expenses, each manufacturing business should ship to each warehouse. The quantity of units to be transported from a given source to a specified destination is directly correlated with the shipping cost. Therefore, in the context of transportation problems, the purpose of the revamped study is to discover a solution that minimizes costs while approaching optimality or achieving it altogether. This is achieved through a thorough analysis of various statistical techniques to determine the most effective approach.

Variables, Objective Function, and Mathematical Formulation of the Model

Let's say a business has m sources and n destinations. The merchandise will be transported from source m to destination n. There is a specific supply level in every warehouse or production establishment and a specific demand in every retail sector. The pair of origin/source and destination has its transport cost for homogenous products indicated as well; these costs are considered linear.

Each retailer seeks a specific number of units of the commodities, as determined by the c_j (j = 1, 2, 3, …, m) store, and each warehouse is only equipped to supply a particular quantity, as determined by the d_k (k = 1, 2, 3, …, n). Transporting an item from j^{th} origin to k^{th} destination is known to cost W_{jk}, and this cost is the same for all combinations (jk).

The objective of the model is identifying the unknown X_{jk}, when all supply and demand constraints are met, will minimize the overall cost of transportation. The aggregate shipping cost for any origin-destination pair combination can now be found by accumulating all the j's and k's. It is feasible to define the transit problem as a linear programming problem model with m, n non-negative constraints. The problem may be expressed mathematically in the following fashion.

Minimization of: $T = \sum_{j=1}^{m}\left[\sum_{k=1}^{n} W_{jk}X_{jk}\right].$

Subject to: $\sum_{k=1}^{n} X_{jk} \leq c_j,\ j = 1,2,3,\ldots,m\ (Supply)$

$\sum_{j=1}^{m} X_{jk} \leq d_k,\ k = 1,2,3,\ldots,n\ (demand)$

$$X_{jk} \geq 0, \quad \forall\, j, k.$$

In this case, d_k = Demand of k^{th} destination (in tons, pounds, liters etc.) and c_j = Capacity of j^{th} source (in tons, pounds, liters, etc.).

A necessary and sufficient conditions for an acceptable solution to the transportation problem to exist is that: $\sum_{j=1}^{m} c_j = \sum_{k=1}^{n} d_k$

which describes the balanced problem for transit problems.

If: $\sum_{j=1}^{m} c_j \neq \sum_{k=1}^{n} d_k$

Is the unbalanced problem, there are again two conditions

i. $\sum_{j=1}^{m} c_j > \sum_{k=1}^{n} d_k$

ii. $\sum_{j=1}^{m} c_j < \sum_{k=1}^{n} d_k$

These problems can be visualized graphically as a set of m, n "directed arcs" and a network with m origins and n destinations. Figure 1 illustrates this network of transportation problems.

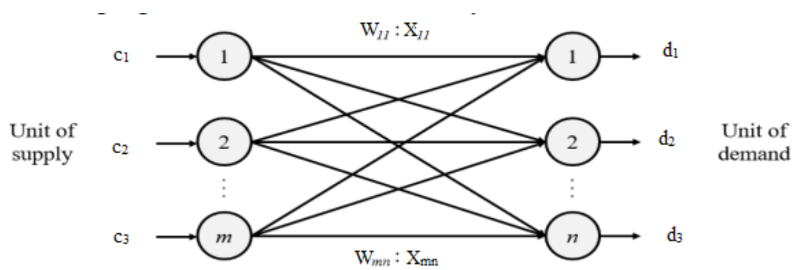

Figure 1: The network of the transit problem with source and destination nodes (Baloch et al., 2022).

Degradation in the Transportation Problems

When the quantity of occupied cells is less than the rows plus columns minus one *(m + n - 1)*, the transit problem is identified as degraded. When the Stepping Stone approach is used, deterioration can be seen during the first allotment when the first item in a row/column fits the row and column demands, or when the increased and removed quantities are equal. The logistics problem with m origins and n destinations is reflected degraded when there are no positive base variables *(m + n - 1).* The transport difficulty is reduced whenever the number of base cells is not greater than or equal to *(m + n - 1)*. To meet the fundamental variable *(m + n - 1),* many zero variables are added to the positive variables to address the degradation problem.

Algorithms of Initial Basic Feasible Solution Methods

Different approaches are used in linear programming to solve, analyse, and compare the results of transit problems with the most accurate methods to determine validity. Some are discussed below.

Vogel Approximation Method (VAM)

The trial-and-error VAM typically yields a better primary solution than other available methods. Using Vogel's' Approximation Method to solve a problem does not ensure that the most effective solution will be found. But its outcome is always achieved with relatively little effort (Hitchcock, 1941).

North-West Corner Method (NWCM)

A technique for calculating the first workable solution to the transportation problem is the NWCM. This approach got its name since the fundamental variables are chosen from the far-left corner (Hitchcock, 1941).

Least Cost Method (LCM)

Taking into consideration the cost of transportation during the allocation, LCM is a more reliable method than NWCM. Wherein the cell with the lowest transit cost serves as the starting point for the allocation. Targeting the least expensive paths allows the LCM to arrive to a better first solution. It gives the cell with the lowest unit cost as much credit as it can. Once a row or column is satisfied, it is marked as crossed out, and the corresponding supply and demand quantities are adjusted accordingly. Similarly, to NWCM, if both a row and a column are satisfied simultaneously, only one of them is marked as crossed out. Subsequently, select the uncrossed cell with the lowest unit cost and repeat the process until exactly one row or column remains uncrossed (Hitchcock, 1941).

For transportation problems, each of the three techniques is productive, however, Vogel's Approximation method (VAM) is more effective as it enhances the initial feasible option. Numerous scholars have proposed adjustments to the present methods in their study by creating unique strategies to reduce the overall cost of transportation. Nonetheless, several techniques have yielded identical outcomes to the LCM, NWCM, and VAM, and some continue to outperform all three techniques in IBFS.

Optimal Method

In the context of transit problem solving, the feasible solution can't be regarded as the best outcome because there may be better solutions (referred to as Basic Feasible Solutions) that optimize the objective function more effectively. The optimality test is used to determine whether a given solution is the Basic Feasible Solution for a given problem and to make any necessary improvements. The optimality is tested using the Stepping-Stone or Modified Distribution (MODI) approach (Vats & Singh, 2016).

Revamped Vogel's Approximation Method (RVAM)

The operational steps below are used to compute the solution for a basic feasible solution.

Step1: Articulate the balanced problem and arrange the data in a matrix. The entire cost of transportation serves as an objective function. The constraints pertain to the supply from each source and the demand at each destination.

Step2: Compute the row and column penalty using any one of the means such as Geometric mean, Arithmetic mean, Contra harmonic mean, and Heronian mean.

Arithmetic Mean: $AM = \bar{X} = \frac{\sum x_j}{n}$,

Geometric Mean: $GM = \left(\prod_j^m X_j\right)^{\frac{1}{m}}$,

Contra Harmonic Mean: $CHM = \frac{X_1^2+X_2^2+X_3^2+\cdots+X_m^2}{X_1+X_2+X_3+\cdots+X_m}$,

Heronian Mean: $HM = \frac{1}{3}(2A\,M + GM)$.

Step3: Pick out the biggest penalty in a row or column computed using each means separately and allocate it to the lowest cost cell.

Step4: Discard the whole row or column in which there is zero supply or demand.

Step5: After all supply and demand have reached zero, terminate the iterations; if not, continue step 2 until a feasible solution is found.

Results of the Transportation Problems

The transportation problem, which deals with transitioning commodities from sources to destinations, is a linear programming problem of the distribution type. The problem is referred to as minimizing if reducing costs is the main goal. Thus, Table 1 to Table 6 below demonstrate the transit problem and the results have been obtained using the different means. Furthermore, findings are also contrasted using linear programming traditional methods including the VAM, NWCM, and LCM are illustrated in Table 7 to Table 10. Assuming x's signifies the source and y's designates the destination.

Transportation Problem 1 (Computing Transportation Costs Using Various Means)

Consider a mathematical model for transit problem *(4×4)*, given as follows (Ullah et al., 2016):

Table 1: Transportation Problem 1 (4×4).

Source/Destination	D1	D2	D3	D4	Supply	AM	GM	CHM	HM
S1	6	3	8	7	110	6	5.6	6.6	5.9
S2	8	5	2	4	60	4.8	4.2	5.7	4.6
S3	4	9	8	4	54	6.3	5.8	7.1	6.1
S4	7	8	5	6	30	6.5	6.4	6.7	6.5
Demand	20	70	78	86	254				
AM	6.3	6.3	5.8	5.3					
GM	6.1	5.7	5	5.1					
CHM	6.6	7.2	6.8	5.6					
HM	6.2	6.1	5.5	5.2					

Now, the balanced transportation problem *(4×4)* presented in Table 1 is solved by various means, respectively as mentioned in Table 2. Therefore, the goal is to reduce the cost of transportation while distributing the commodities to the four destinations from the four suppliers. Employing the proposed method, initially, indicators for each row and column are calculated using means respectively (see Table 2). Selecting the largest value of penalty in a row or column and allocating it to small cost cells, and then, deleting the row or column that becomes zero, then developing a new allocation table. The process continues until all cells representing supply and demand are fulfilled.

Table 2: Penalty Distribution by various Means.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6	3	8	7	110
S2	8	5	2	4	60
S3	4	9	8	4	54
S4	7	8	5	6	30
Demand	20	70	78	86	$\sum_{j=1}^{m} c_j = \sum_{k=1}^{n} d_k$

Choosing the highest penalty that is 6.5 obtained using arithmetic mean in Table 2 and selecting the small cost in that row $W_{43}=5$. Here, we assign $X_{43} = min\ (c_4, d_3) = min\ (30, 78) = 30$. Satisfying the demand and supply and deleting the chosen row that has become zero. Continue the same process and we obtained the summarized results demonstrated in Table 3.

Table 3: Summarized Balanced Transportation Problem by Arithmetic Mean.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6	3⌋70	8	7⌋40	110
S2	8	5	2⌋48	4⌋12	60
S3	4⌋20	9	8	4⌋34	54
S4	7	8	5⌋30	6	30
Demand	20	70	78	86	254

Finally, transportation cost is calculated by weighing the objective function as follows,

$Min\ T = W_{12}X_{12} + W_{14}X_{14} + W_{23}X_{23} + W_{24}X_{24} + W_{31}X_{31} + W_{34}X_{34} + W_{43}X_{43},$

$= 3\times70 + 7\times40 + 2\times48 + 4\times12 + 4\times20 + 4\times34 + 5\times30,$

$= 210 + 280 + 96 + 48 + 80 + 136 + 150 = 1000.$

In a similar manner, Table 4 to Table 6 are constructed for the balanced transportation problem 1 for minimizing the cost using the geometric mean, contra-harmonic mean, and Heronian mean respectively. While the Table 7 to Table 9 are tailored for the traditional methods.

Table 4: Balanced Transportation Problem by Geometric Mean.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6	3⌋70	8	7⌋40	110
S2	8	5	2⌋48	4⌋12	60
S3	4⌋20	9	8	4⌋34	54
S4	7	8	5⌋30	6	30
Demand	20	70	78	86	254

$Min\ T = W_{12}X_{12} + W_{14}X_{14} + W_{23}X_{23} + W_{24}X_{24} + W_{31}X_{31} + W_{34}X_{34} + W_{43}X_{43},$

$= 3\times70 + 7\times40 + 2\times48 + 4\times12 + 4\times20 + 4\times34 + 5\times30,$

$= 210 + 280 + 96 + 48 + 80 + 136 + 150 = 1000.$

Table 5: Balanced Transportation Problem by Contra Harmonic Mean.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6⌋20	3⌋70	8	7⌋20	110
S2	8	5	2⌋60	4	60
S3	4	9	8	4⌋54	54
S4	7	8	5⌋18	6⌋12	30
Demand	20	70	78	86	254

$Min\ T = W_{11}X_{11} + W_{12}X_{12} + W_{14}X_{14} + W_{23}X_{23} + W_{34}X_{34} + W_{43}X_{43} + W_{44}X_{44},$

$= 6\times20 + 3\times70 + 7\times20 + 2\times60 + 4\times54 + 5\times18 + 6\times12,$

$= 120 + 210 + 140 + 120 + 216 + 90 + 72 = 968.$

Table 6: Balanced Transportation Problem by Heronian Mean.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6	3⌋70	8	7⌋40	110
S2	8	5	2⌋48	4⌋12	60
S3	4⌋20	9	8	4⌋34	54
S4	7	8	5⌋30	6	30
Demand	20	70	78	86	254

$$Min\ T = W_{11}X_{11} + W_{12}X_{12} + W_{13}X_{13} + W_{23}X_{23} + W_{34}X_{34} + W_{43}X_{43} + W_{44}X_{44},$$
$$= 3\times70 + 7\times40 + 2\times48 + 4\times12 + 4\times20 + 4\times34 + 5\times30,$$
$$= 210 + 280 + 96 + 48 + 80 + 136 + 150 = 1000.$$

Table 7: Balanced Transportation Problem by Vogel's Approximation Method.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6⌋20	3⌋70	8	7⌋20	110
S2	8	5	2⌋60	4	60
S3	4	9	8	4⌋54	54
S4	7	8	5⌋18	6⌋12	30
Demand	20	70	78	86	254

$$Min\ T = W_{11}X_{11} + W_{12}X_{12} + W_{14}X_{14} + W_{23}X_{23} + W_{34}X_{34} + W_{43}X_{43} + W_{44}X_{44},$$
$$= 6\times20 + 3\times70 + 7\times20 + 2\times60 + 4\times54 + 5\times18 + 6\times12,$$
$$= 120 + 210 + 140 + 120 + 216 + 90 + 72 = 968.$$

Table 8: Balanced Transportation Problem by North West Corner Method.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6⌋20	3⌋70	8⌋20	7	110
S2	8	5	2⌋58	4⌋2	60
S3	4	9	8	4⌋54	54
S4	7	8	5	6⌋30	30
Demand	20	70	78	86	254

$$Min\ T = W_{11}X_{11} + W_{12}X_{12} + W_{13}X_{13} + W_{23}X_{23} + W_{24}X_{24} + W_{34}X_{34} + W_{44}X_{44},$$
$$= 6\times20 + 3\times70 + 8\times20 + 2\times58 + 4\times2 + 4\times54 + 6\times30,$$
$$= 120 + 210 + 160 + 116 + 8 + 216 + 180 = 1010.$$

Table 9: Balanced Transportation Problem by Least Cost Method.

Origin	Destinations				Supply
Source	D1	D2	D3	D4	
S1	6⌋20	3⌋70	8	7⌋20	110
S2	8	5	2⌋60	4	60
S3	4	9	8	4⌋54	54
S4	7	8	5⌋18	6⌋12	30
Demand	20	70	78	86	254

$Min\ T = W_{11}X_{11} + W_{12}X_{12} + W_{14}X_{14} + W_{23}X_{23} + W_{34}X_{34} + W_{43}X_{43} + W_{44}X_{44}$
$= 6\times20 + 3\times70 + 7\times20 + 2\times60 + 4\times54 + 5\times18 + 6\times12,$
$= 120 + 210 + 140 + 120 + 216 + 90 + 72 = 968.$

The solution for the transportation cost problem of Table 1 has been achieved by proposed as well as conventional methods. The optimal solution by utilizing MODI for the transit problem 1 is 968. It is noteworthy here that the aggregate minimum cost specified by objective function using CH, VAM, LCM, given as: $6\times20 + 3\times70 + 7\times20 + 2\times60 + 4\times54 + 5\times18 + 6\times12 = 968$ is coherent to the optimal solution. In addition, Table 10 identified that the proposed method (contra harmonic mean) surpassed all other traditional approaches in attaining the initial basic feasible solution that also matches with the existing literature (Ullah et al., 2016). The juxtaposed results described in Table 10 has also been illustrates in Figure 2.

Table 10: Comparison of the revamped model with the conventional models for TP 01.

Methods	Initial Basic Feasible Solution	Optimal Solution
North-West Corner	1010	
Least Cost method	968	
Vogel's Approximation	968	
Contra Harmonic Mean	968	968
Arithmetic Mean	1000	
Geometric Mean	1000	
Heronian Mean	1000	
Published Literature	968	

Problem 2 to problem 5 related to Transportation are calculated using both traditional and offered models and problems are presented in Table 11. The correlation of the findings with the traditional methods and the published literature are scanned in this research to analyse the efficacy of the suggested method (see Table 12).

Table 11: Transportation Problems.

Transportation Problem 2: Assuming a (3×4) (Ullah et al., 2016)						Transportation Problem 3: Assuming a (4×5) (Das et al., 2014)						
Source/Dest.	D1	D2	D3	D4	Supply	Source/Dest.	D1	D2	D3	D4	D5	Supply
S1	4	5	8	4	52	S1	4	4	9	8	13	100
S2	6	2	8	1	57	S2	7	9	8	10	4	80
S3	8	7	9	10	54	S3	9	3	7	10	6	70
Demand	60	45	8	50	163	S4	11	4	8	3	9	90
						Demand	60	40	100	50	90	340
Transportation Problem 4: Assuming a (3×4) (Soomro et al., 2015)						Transportation Problem 5: Assuming a (3×4) (Ackora-Prah et al., 2023; Sivakumara et al., 2015)						
Source/Dest.	D1	D2	D3	D4	Supply	Source/Dest.	D1	D2	D3	D4	Supply	
S1	20	22	17	4	120	S1	11	13	17	14	250	
S2	24	37	9	7	70	S2	16	18	14	10	300	
S3	32	37	20	15	50	S3	21	24	13	10	400	
Demand	60	40	30	110	240	Demand	200	225	275	250	950	

Table 12: Comparative Results of Transportation Problems.

Methods	Example 2	Example 3	Example 4	Example 5
North-West Corner	914	2490	3680	12200
Least Cost method	674	1710	3580	12200
Vogel's Approximation	674	1710	3520	12075
Arithmetic Mean	674	1690	3460	12075
Heronian Mean	674	1670	3460	12075
Geometric Mean	674	1840	3460	12075
Contra Harmonic Mean	674	1670	3460	12075
Published Literature	674	1670	3460	12075
Optimal Solution (MODI)	674	1670	3460	12075

Graphical Representation of the Results

The juxtaposed results reported in Table 12 are also illustrated using bar graphs and the findings are displayed in Figure 3 to Figure 6. It is plainly evident from Table 12 and Figure 3, Figure 5, and Figure 6, that the proposed technique is more straightforward to use, better-performing, and accurately optimized for the most feasible outcome. Furthermore, small deviation in the aggregate cost from optimal result is witnessed in Arithmetic-Geometric mean for problem 3 illustrated Figure 4, whereas contraharmonic mean and heronian mean reaching to the optimal solution. Moreover, a comparison with traditional methods reveals that the results obtained from the revamped method fully match the optimal results as well as the existing literature.

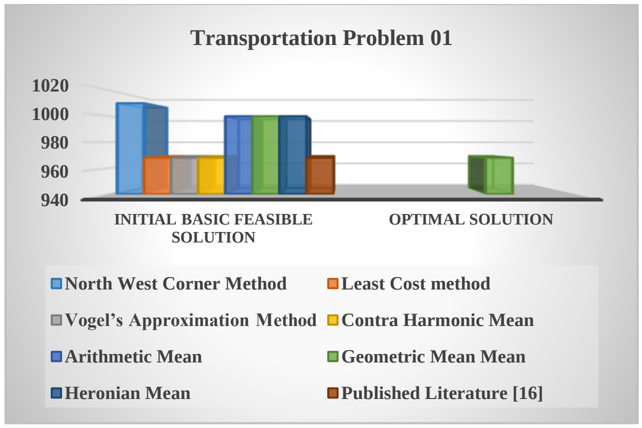

Figure 2: Comparative Study of TP 01 (4×4).

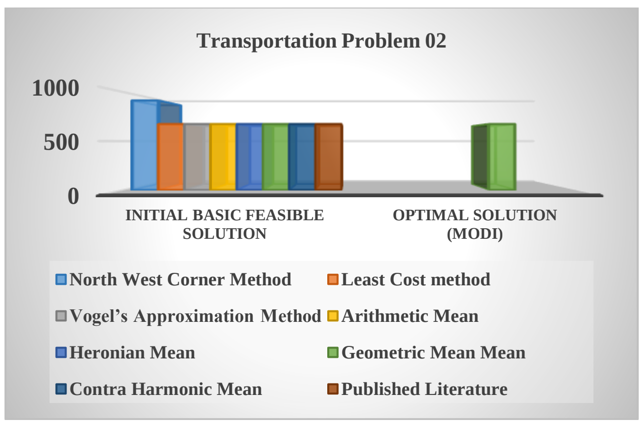

Figure 3: Comparative Study of TP 02 (3×4).

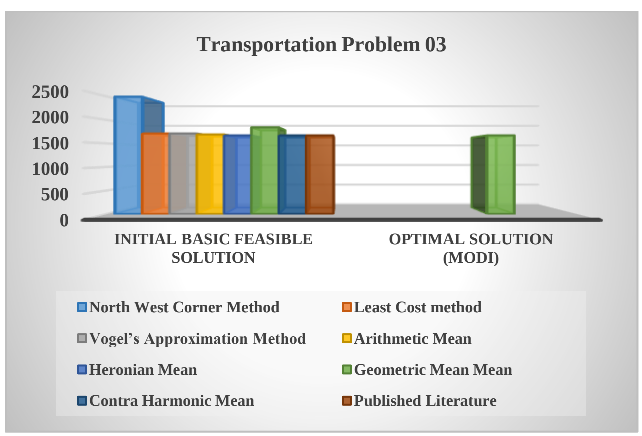

Figure 4: Comparative Study of TP 03 (4×5).

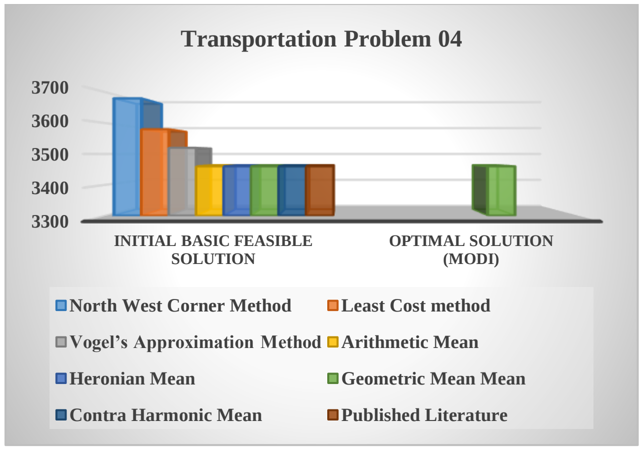

Figure 5: Comparative Study of TP 04 (3 ×4).

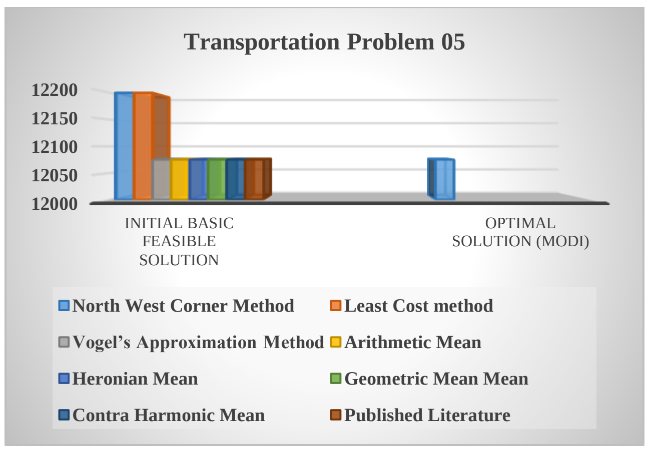

Figure 6: Comparative Study of TP 05 (3 ×4).

Conclusion

This study signifies a novel approach to the transportation problem to identify the basic solution. The conventional Vogel's approach is renowned for its ability to provide crucial solutions to problems pertaining to transportation. Nonetheless, the goal of this study is to refurbish VAM by statistical technique to effectively handle certain transportation problems. Some examples were tested, and the verdicts are presented in the tables. We examined the effectiveness of the suggested approach by comparing the solution with the significant solutions found in other published studies as well as traditional approaches. The results are also subsequently juxtaposed with the optimal solution derived from the MODI method. It is comprehended that the results acquired by RVAM is very close to optimal or equal to optimal, surpassing those of traditional methods such as NWCM, VAM, and LCM.

References

Abdul Sattar Soomro, Muhammad Junaid, Gurudeo Anand Tularam, (2015), Modified Vogel's Approximation method for solving Transportation problem, *Mathematical Theory and Modeling,* 5(4).

Ackora-Prah, J., Acheson, V., Owusu-Ansah, E., & Nkrumah, S. K. (2023). A Proposed Method for Finding Initial Solutions to Transportation Problems. *Pakistan Journal of Statistics and Operation Research*, 63-75.

Amaliah, B., Fatichah, C., & Suryani, E. (2019). Total opportunity cost matrix–Minimal total: A new approach to determine initial basic feasible solution of a transportation problem. *Egyptian Informatics Journal*, *20*(2), 131-141.

Amaliah, B., Fatichah, C., & Suryani, E. (2022). A supply selection method for better feasible solution of balanced transportation problem. *Expert Systems with Applications*, *203*, 117399.

Babu, M. A., Hoque, M. A., & Uddin, M. S. (2020). A heuristic for obtaining better initial feasible solution to the transportation problem. *Opsearch*, *57*(1), 221-245.

Baloch, N., Shaikh, A. A., Shaikh, W. A., & Qureshi, S. (2022). Balanced Transportation Problems with an Alternative Methodology for the Allocation Table Method. *Statistics, Computing and Interdisciplinary Research*, *4*(1), 99-111.

Das, U. K., Babu, M. A., Khan, A. R., & Uddin, M. S. (2014). Advanced Vogel's Approximation Method (AVAM): a new approach to determine penalty cost for better feasible solution of transportation problem. *International Journal of Engineering Research & Technology (IJERT)*, *3*(1), 182-187.

Eiselt, H. A., & Sandblom, C. L. (2022). *Operations research: A model-based approach*. Springer Nature Switzerland AG, June 17, 2022

Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of mathematics and physics*, *20*(1-4), 224-230.

Hosseini, E. (2017). Three new methods to find initial basic feasible solution of transportation problems. *Applied Mathematical Sciences*, *11*(37), 1803-1814.

Hossain, M. M., & Ahmed, M. M. (2020). A comparative study of initial basic feasible solution by a least cost mean method (lcmm) of transportation problem. *American journal of operations research*, *10*(04), 122.

Juman, Z. A. M. S., & Hoque, M. A. (2015). An efficient heuristic to obtain a better initial feasible solution to the transportation problem. *Applied Soft Computing*, *34*, 813-826.

Jain, S. (2015). The maximum difference method to find initial basic feasible solution for transportation problem. *Asian Journal of Management*, 8.

Karagul, K., & Sahin, Y. (2020). A novel approximation method to obtain initial basic feasible solution of transportation problem. *Journal of King Saud University-Engineering Sciences*, *32*(3), 211-218.

Korukoğlu, S., & Ballı, S. (2011). An improved Vogel's approximation method for the transportation problem. *Mathematical and Computational Applications*, *16*(2), 370-381.

Kumar, R., Gupta, R., & Karthiyayini, O. (2018). A new approach to find the initial basic feasible solution of a transportation problem. *Int. J. Res. Granthaalayah*, *6*(5), 321-325.

Mallick, C., Bhoi, S. K., Singh, T., Swain, P., Ruskhan, B., Hussain, K., & Sahoo, K. S. (2023). Transportation Problem Solver for Drug Delivery in Pharmaceutical Companies using Steppingstone Method. *International Journal of Intelligent Systems and Applications in Engineering*, *11*(5s), 343-352.

M. V. Crankson, H. Otoo, P. K. Nyarko, H. Cobbina, (2023). Modified Vogel's Approximation Method for Finding Optimal Solution of Transportation Problem: A Case Study at a Mining Company, *American Journal of Computational and Applied Mathematics*, 13(1). 15-20.

Sivakumara, J. G., & Priyab, S. (2015). RC-MMM method for finding an optimal solution for transportation problem. *Malaya Journal of Matematik (MJM)*, (1, 2015), 82-88.

Ullah, M. W., Uddin, M. A., & Kawser, R. (2016). A modified Vogel's approximation method for obtaining a good primal solution of transportation problems. *Annals of Pure and Applied Mathematics*, *11*(1), 63-71.

Vats, B., & Kumar Singh, A. (2016). Solving transportation problem using excel solver for an optimal solution. *MIT International Journal of Mechanical Engineering*, *6*(1), 18-20.