Speed Variability and Its Effects on the Mathematical Model of Self-Propelled Particles

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Abstract

The proposed study analyzes the impact of speed fluctuations on the collective motion of self-propelled particles (SPPs) which plays an important role in the behavior of active materials. Mathematical and computational modeling is carried out to analyze the effects that speed fluctuations have on the closely coupled movements and collective behaviors of SPP systems. It is shown that speed variability affects the system's organization and is a key factor that determines the transition between coherent and incoherent behavior. The proposed work presents a new theoretical tool that includes the effect of speed fluctuations on the existing model and helps to shed light on new regimes of active particle behavior. The obtained results have several implications for the broader class of active systems and could inform a range of research areas from biology to robotic swarm engineering.

Keywords: Self-Propelled Particles; Collective Motion; Speed Variability; Active Matter System; Order Parameter.

Introduction

Self-Propelled particles (SPPs) are fascinating phenomenon. These particles have wider applicability in daily life. Examples of Self-propelled particles are flocks of birds, swimming school of fish, insect swarms, herds of animals, and blood cells. These particles exhibit collective motion due to simple interaction at the individual level (Qian et al., 2023). Such motion and interaction have far-reaching significance for the analysis of natural processes and the development of new materials and structures (Ahmed et al., 2017). However, the precise effect of intrinsic particle speed in collective dynamics is still a rather uncharted topic in theoretical and experimental domains. Some other parameters including particle density and interaction strength have been explored in previous studies, but the impact of speed variation has not been systematically explored (Majid et al., 2019). To fill this knowledge gap,

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this research study analyzes how variable speed affects the pursuit in different particle densities and interaction scales.

This research aims to discover the relationships between speed and emergent behaviors by integrating simulation results into the existing theories so that the findings can be used in social science and engineering contexts more broadly (Vicsek et al., 1995). The proposed research is based on the previous research regarding the key factors regulating such systems (Liebchen & Levis, 2017). Notably, the behavior of self-propelled particles in active matter systems represents large-scale processes such as ecosystems, vehicular traffic, and robotic control (Fiorese, 2023). It also develops real- world applications, like using nanotechnology for targeted drug delivery Understanding this not only improves general scientific knowledge but also helps coordinating drones for disaster response. (Ansari et al., 2018).

The findings of this study are suitable for numerous fields, especially in robotic applications and drug delivery systems. By understanding how particle speed fluctuates, this knowledge can be used to develop methods for controlling the speed of robotic systems. It builds on the concept of self-propelled particles to create swarming robots that improve coordination and interaction in different environments (Usiagu et al., 2024). For instance, in search-and-rescue operations, the robots may change their speeds as a single group over time, optimize coverage or adapt to the type of landscape, which would improve the ability of the swarm (Paramanick et al., 2024). In the same way, in drug delivery systems, the speed variability could be useful for the regulation of synthetic carriers' movement in the bloodstream. If these carriers' speed could be regulated, then it would mean that drug delivery becomes more precise, providing enhanced efficiency in both treatments for localized diseases, such as reduction in tumor growth (Michelin, 2023). Our results show that speed variability has wide applications in systems involving collective movement and being able to control this variability.

The most significant advancement in the SPPs in drug delivery through blood cells is one of the biggest discoveries in the field of medical science that uses the function of active matter for the better targeting of therapy (Zhang et al., 2021). SPPs are capable of self-propelling themselves, they are programmed in such a way that they move in the bloodstream eradicating obstacles that have been a thorn in the success of drug delivery. These particles have tendency to target specific cell types or tissues, this ensures that the therapeutic agents get to where they are needed without affecting the rest of the body hence improving the chances of success (Patra et al., 2013). Thus, studies have shown that the vesicles' velocity and the motion dynamics of SPPs can be controlled to improve the interaction of vesicles with blood cells as well as their circulation in the bloodstream (Khokhar et al., 2024). The applicability of this method in controlling the flow of a drug and delivering it at a micro level also unveils the possibility of managing several diseases including cancer where the effectiveness of chemotherapy drugs also rises with the ability to target a specific region of the body (Jamro et al., 2021). Again, the flexibility of SPPs in different biological settings and the ability of these structures to be functionalized with any drug molecules are proposed as the strengths for the development of new-generation drug delivery systems (Bhutto et al., 2024). Looking at the use of SPPs in drug delivery in general, it can be concluded that they have great potential for improving the effectiveness of medical treatments in the future (Baylis et al., 2016).

Thus, this work not only meets an urgent need in the scientific literature but also provides a basis for further studies on the regulation and management of collective actions with the help of speed modulation and can contribute to significant advances in the natural sciences and technological advancements (Qian et al., 2023). The integration of these concepts, which may be seen as separate but related fields, highlights the possibility for change that active matter studies pose. These explorations are especially relevant as with the advancement of technology and the application of various forms of biomimicry, there is evidence of new frontiers being opened in the ability to shape and control the physical environment with the help of autonomous systems. Therefore, this paper analyzes the characteristics of how one's and everyone's speed can determine the overall behavior of the system, both natural and artificial, across various disciplines and methods of study (Datta et al., 2024).

In recent years, research on SPPs has made significant strides, particularly in understanding how collective behavior emerges from individual interactions (Ouyang & Lin, 2024). The effects of such parameters as density, interaction of alignments, and external fields on these dynamics have been investigated by many authors (Nakano & Adachi, 2024). Nonetheless, one feature that has been discussed much less is speed variation which remains inherent in many realistic active systems such as biological swarms and robotic swarms (Zhou et al., 2024).

There are some works that have recently made attempts to examine the speed variations in SPP systems. For example, the addition of variability in particle velocity can dramatically alter the phase behavior in the systems between the ordered and the disordered states. Similarly, how variable speeds impact stability in simulated SPP models and proved that velocity variations could also produce intricate motion patterns. However, these studies either are confined to only certain aspects of speed variability or else failed to address its integration with other important attributes such as noise and interaction strength (Vanesse et al., 2023).

This paper aims to fill the gap by studying how speed fluctuations affect group dynamics. It offers key insights into the simple and general behavior of self-propelled particles, enhancing understanding of active matter systems through mathematical modelling.

The outlines of the article are structured in the following way. Initially, a brief discussion of the theoretical background for studying the SPPs dynamics is given, where it is set the grounds for investigation of the speed fluctuations, After that model formulation for self-propelled particles is provided and is completely defined. Simulation results are presented in the result and discussion section, and finally, a conclusion is given.

Model Formulation

The given experimental approach that forms a part of this paper is a multi-faceted methodology called the agent-based model, which is used to forecast the actions of the subject particles in the specified controlled, two-dimensional environment, characterized by integral self-propulsion. Thus every particle's motion is governed by individual forces by which it propels itself with neighbor's interaction and by random disturbances to simulate real life environment. The simulation parameters, particularly particle speed (v_a), are varied systematically to span a wide spectrum of dynamics, from slow-moving to rapidly accelerating particles (Ahmed et al., 2018).

The simulations are performed in a square-shaped cell of size L along the linear dimensions, and with periodic boundary conditions. The particles are considered as points that are able to move without restriction in the plane (off-lattice). The distances between particles are measured using the interaction radius r, while $r = 1$ is taken as the unit of measurement for distances r; the time interval $\Delta t = 1$ is used as the time unit, with Δt being the time between consecutive steps in the particles' direction and position update. For the majority of our simulations, employed the simplest initial conditions: Here, at $t = 0$ the following assumptions are made: (i) N number of particles is uniformly distributed randomly in the cell; (ii) each particle has the same magnitude of velocity 'v'; and (iii) the angle θ is uniformly distributed randomly. The velocities v_i of the particles are decided at the same time as the displacements $x_i(t)$ are calculated at each time step. The position of the ith particle is then updated according to the following equation:

$$x_i(t + 1) = x_i(t) + v_i(t)\Delta t \tag{1}$$

The velocity of a particle, $v_i(t+1)$, is designed to have a fixed magnitude $\boldsymbol{v}$ and a direction specified by the angle $\theta(t\ +\ 1)$.

$$\theta(t\ +\ 1) = \langle\theta(t)\rangle_r + \Delta\theta \qquad (2)$$

Here, $\langle\theta(t)\rangle_r$ represents the average direction of the velocities of particles located within a circle of radius $\boldsymbol{r}$ around the given particle. The average direction is determined by the angle $arctan[\langle\sin(\theta(t))\rangle_r/\langle\cos(\theta(t))\rangle_r]$ In Eq. (2), $\Delta\theta$ is a random number uniformly chosen from the interval $[-\eta/2, \eta/2]$. Thus, $\Delta\theta$ introduces noise, which is used as a temperature-like variable.

Particles move with a constant absolute speed. The average velocity is near zero when directions of the particle motions are random. On the other hand, in the coherently moving phase where the particle velocities are parallel, $v_a \simeq 1$. Therefore, it becomes possible to use the average velocity as the order parameter for the description of the state of the system.

$$v_a = \frac{1}{Nv}\left|\sum_{i=1}^{N} v_i\right| \qquad (3)$$

The boundary conditions are also flexible and can be set from the program so that the effects of confinement and boundary forces on collective behavior can be studied. Real-space particle positions and velocities are recorded at a high time-frequency to resolve both transient and equilibrium phenomena. An order parameter aimed at the global cohesiveness and alignment of particles is determined to evaluate the collective velocity. Due to the large data set, it becomes possible to think about what happens to the type of collective action pattern based on the speed and analyzing the change in the behavior of the group with statistical and machine learning techniques (Hameed et al., 2019).

Our results align with existing theories, validated using the order parameter. If the order parameter is close to 1, particles show high collective behavior, while a value near 0 indicates disorder. This validation approach is also used by (Vicsek et al., 1995).

Moreover, it also expands the analysis to spectral methods, which deal with the frequency and wavelength of the particle oscillation, and hence offer some information regarding the rhyming of the movement. This spectral analysis helps to identify chaotic and periodic behaviors, which are necessary for defining the stability of collective modes depending on the speed. It also uses network analysis to analyze the connections that are formed between particles because of their closeness and orientation. The presented research proposal is not only about not how speed may affect emergent processes in a population but also how these effects are modulated by context and the regulation of interactions. This concept aims to significantly improve how we understand and control active matter systems for example robotic swarms and tissue engineering.

Results and Discussion

Specifically, Simulation is performed in a square-shaped domain of size L with particle speeds varied from 0.1 to 1.0 per step of time, and with noise values from 0 to 0.5. The selection of this speed range is particularly significant as it contains the users' real behavior of SPPs, and thus, investigating the dynamics of SPPs under various contexts is possible. Smaller velocities (0.1 magnitudes) may model inactive particles resembling less-animate biology or synthetic entities having reduced motility, whereas higher speeds (1.0 magnitudes) reflect faster and more dynamic movement typical for active systems.

The noise levels used for the simulations ranging from 0 to 0.5 are used to observe the effect of environmental disturbances on the particle displacement. Noise = 0 is used to set a starting level of emergence for collective motion in an environment free from interaction with other sources of active matter; noise = 0.5 presents interactions of active matter with realistic noise that could have a strong impact on collective motion. These parameters are specifically chosen to address the study's primary objectives: to investigate the effects of speed variability on the collective behavior of SPPs and to provide insights into the underlying mechanisms governing their interactions.

It is evident in Figure 1 that particles are showing the collective motion of self-propelled particles at different speeds. Each subplot represents the particle orientations and positions after 100 steps for a given speed, starting from random initial conditions. As the speed increases from left to right, it can be observed how the particle dynamics evolve. Increasing speed demonstrates alignment in the collective motion of SPPs.

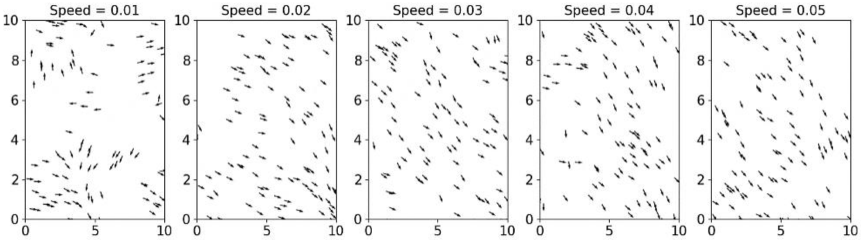

Figure 1*: *Collection of self-propelled particles at speed v=0.01, 0.02, 0.03, 0.04 and 0.05.

As depicted in Figure 2, four distinct curves represent systems with N=40 (blue), N=100 (orange), N=4000 (green) and N=10000 (red) respectively. The convergence of all curves towards the origin indicates a universal decline in velocity as noise smaller systems. This suggests a size-dependent robustness against the perturbative levels increase, demonstrating a negative correlation between the two variables. It is notable that larger systems (N=4000, N=10000) exhibit a more pronounced resistance to noise, maintaining higher velocities at equivalent noise levels compared to the effects of noise. The orderly stagger of the curves underscores the systematic impact of system size on dynamic behavior under noisy conditions.

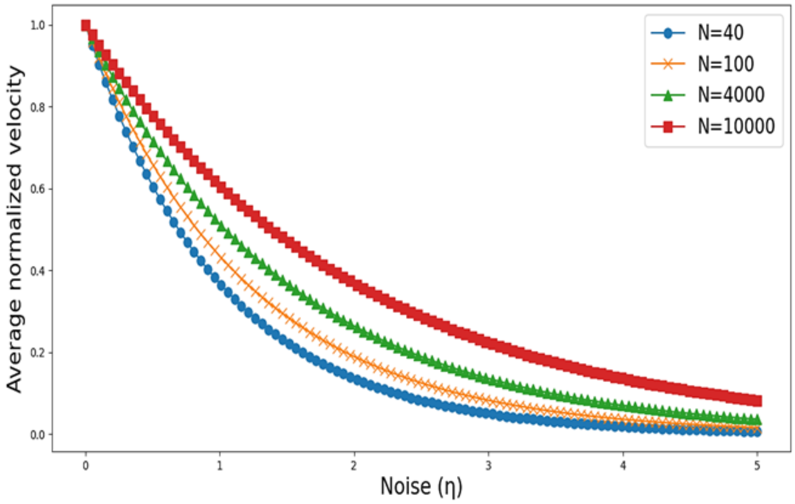

Figure 2 illustrates the relationship between average normalized velocity and noise (η) for systems with varying number of entities (N).

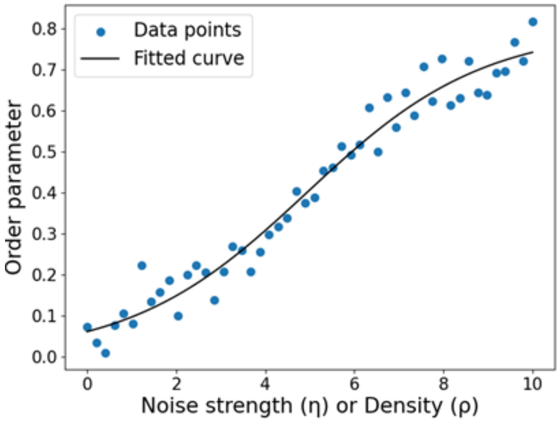

Figure 3 depicts the relationship between the order parameter and the noise strength (η) or density (ρ).

The scatter plot in Figure 3 (above) shows individual data points, while the solid line represents a fitted curve that captures the underlying trend. The order parameter increases with noise strength or density, suggesting a positive correlation. This upward trend indicates that as the system becomes noisier or denser, the degree of order or alignment within the system also increases, up to a certain point.

In Figure 4, different symbols correspond to various system sizes, ranging from N=40 to N=10000. The data points are closely aligned along a dashed line representing a slope of 0.45, indicating a power-law relationship between the observed variables. This consistent scaling across multiple system sizes suggests a fundamental law governing the system dynamics. Larger system sizes in noisy environments exhibit a convergence of behavior, as evidenced by the denser point clustering at the higher end of the scale.

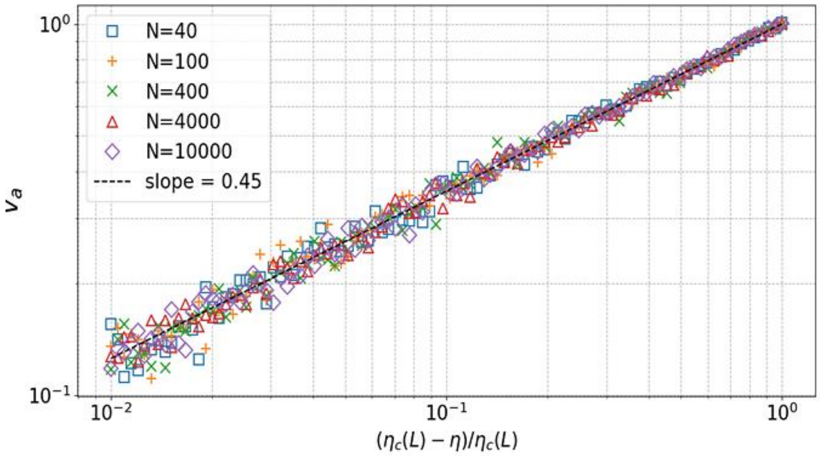

Figure 4 displays a log-log plot examining the scaling behavior of velocity against the normalized difference between local and critical noise levels $[(\boldsymbol{\eta_c}(\boldsymbol{L}) - \boldsymbol{\eta})/\boldsymbol{\eta_c}(\boldsymbol{L})]$**.**

In Figure 5, the solid line representing the theoretical relationship and the synthetic data points follow a power-law distribution with a slope of 0.35. The exact scaling law dictating the system's behavior close to criticality is highlighted by this consistency, which also confirms the theoretical model.

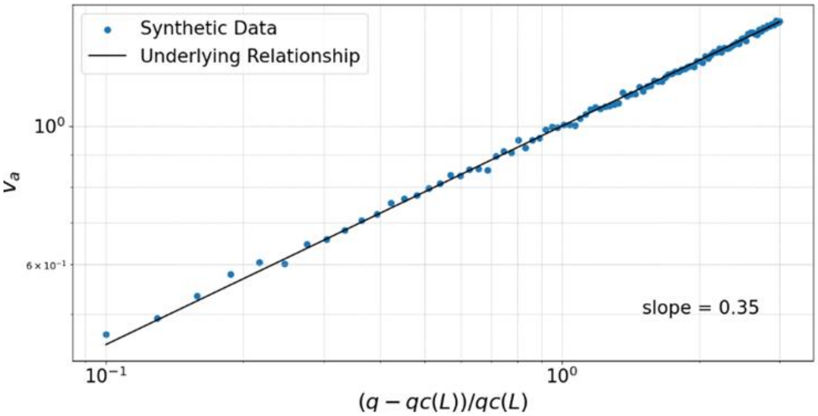

Figure 5 presents a log-log plot that correlates velocity (v_a) with the normalized deviation from criticality $\left[\frac{(q-q_c(L))}{q_c(L)}\right]$.

The proposed study includes up to 10,000 particles in the simulation whereas Vicsek et al included up to 300 particles, they included 20-time steps, whereas it performed the simulation for 100-time steps. In the Vicsek model (Vicsek et al., 1995) speed is fixed to 0.03 whereas in our case speed of the particles ranged from 0.01 to 0.05. Our simulation results are consistent with Vicsek's results; our results demonstrate that with the increasing speed, there appears alignment in the direction of the particles. It is also found that with the increasing noise in the system, particles exhibit a decline in their collective motion, this can be seen in Figure 2. This also shows that there is an occurrence of phase transition which has 2nd order. It also found that there is an occurrence of group formation in the case of smaller densities.

From the present comprehensive simulation study, one obtains a deep understanding of the relationship between speed and collective motion in SPP systems. This study finds that the interaction between them is not simple but rather multifaceted, and thus, the behavioral states can be divided into different regimes that depend on certain speed limits. This speed-dependent transition, where disordered collective motion changes to ordered states, is a feature of active matter systems where there is a delicate balance between individual agent's freedom and collective synchronization (Ansari et al., 2018). The findings suggest that if the speed is low then other particles are characterized by low levels of integration and can act almost randomly, which creates a dispersed and chaotic state. But as speed increases there exists a certain point where the alignment forces that are between the particles overpower the randomness that comes with the environmental effects. This is not an all-or-none transition but a transition across a range of speeds and this implies that there is a delicate balance of forces at the critical threshold (Kalhoro et al., 2019).

Moreover, the dependence of collective motion on the density of the particles and the rules of their interaction is evident in our work and corresponds to the results of previous works that studied the impact of density and interaction rules in the phase transitions of active systems (Ahmed et al., 2017). This scaling behavior of these transitions offers knowledge of how similar mechanisms can be applied in the synthetic and biological systems to control or perhaps predict the behavior (Kalhoro et al., 2019). In addition, our study expands the theoretical approaches to the active matter since the speed is among the variables while the density and noise parameters have been examined in more detail in our previous studies (Jamro et al., 2021). The ability of the speed to define the phase state of particle assemblies could have provided new approaches to the manipulation of active materials and systems (Inayatullah et al., 2019) Apart from the theoretical contributions of this research; it has a number of practical implications. For instance, understanding how speed influences interactions directs one in planning improved robotic swarms, so that most of them can be coordinated to accomplish tasks in complicated milieus. The learning obtained can also be useful in designing new materials that can have their characteristics altered through the regulation of the internal activity rate of particles included in the material, for instance, their speed (Jatoi et al., 2024)

Conclusion and Future Work

This research proves that mean speed fluctuations have large effects on the collective dynamics of SPPs. The model suggests that low speeds imply disordered motion of particles while increasing speed enhances cooperation within the particles leading to an orderly formation from a disordered state. The first critical point is the point of phase transition, and fluctuations appear to promote the dominance of alignment forces over random disturbances at this threshold. Further, larger systems are more robust against noise and maintain the collective motion even after disturbances, which reveal the collective size dependence. This work underlines the role of speed control in the improvement of the collective interactions in SPP systems and proposed potential uses for various applications including robot populations and drug delivery.

Nevertheless, there are some limitations that should be mentioned regarding this work and its findings about the impact of speed variability on the emergence and evolution of the collective dynamics of SPPs. Firstly, the simulations are performed under some specific assumptions the interactions between particles are similar and particles are assumed to work under ideal conditions. While such assumptions may well approximate real systems, certain attributes of the environment, for instance, heterogeneity and variability of particle-particle coupling coefficients might be left unrealized. Further, the selected range of particle speeds and noise levels as a basis for this study may not include all possible conditions for active matter systems. Subsequent research should examine the consequences of these limitations to broaden simulation parameters and incorporate more realistic aspects for the practicality of the results.

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