Analysis and Enhancement of Newton Raphson Method

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***Abstract*:**
Recently, Newton Raphson-based Algorithms 2.1 and 2.2 are developed to solve different nonlinear systems of nonlinear equations. These newly developed algorithms have demonstrated superior performance compared to the traditional Newton-Raphson Method (NRM) and the Fixed Point Theorem. They took less time as compared to these conventional methods by reducing the number of iterations required to converge to an exact solution and thereby decreasing the computational work. It is also observed that these performance measures can be further improved. Accordingly, in this research, we present some efficient numerical algorithms for solving nonlinear and systems of nonlinear equations based on NRM. The Modified Adomian Decomposition Method (MADM) is applied to construct the numerical algorithm. A new Algorithm 2.3 is developed, an enhancement in previous relevant algorithms. Several nonlinear expressions are estimated via enhancements in NRM. The results of these tests are measured in terms of the number of iterations and presented in the form of tables to show the converging rate to an exact solution. The obtained results of Algorithm 2.3 are compared with other methods like NRM, Fixed Point Method (FPM), Algorithm 2.1, and Algorithm 2.2 to show the efficiency of the suggested enhancement. It is finally observed after solving several nonlinear equations that Algorithm 2.3 is converging more rapidly than other numerical methods. Consequently, Algorithm 2.3 significantly reduces the number of iterations needed to achieve an exact solution as compared to existing other iterative approaches in getting the exact solution. This efficiency makes it a valuable tool for solving nonlinear equations more effectively.

Keywords: Newton Raphson Method; Fixed Point Method; Algorithm 2.1; Algorithm 2.2; Algorithm 2.3.

Introduction

Numerical analysis is a field of mathematics, computer, economics, and other sciences that generates analyses and implements algorithms for getting numerical solutions to problems containing continuous variables. Numerical analysis defines a root for the solution of a simple equation like nonlinear equations. The nonlinear equations are those equations whose graphical solutions don't make straight lines. In the nonlinear equations, the variables are either of degree less or greater than

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one. The equations in the form of polynomials are known as algebraic equations and the equations containing hyperbolic, trigonometric, exponential, or logarithmic functions assumed to be transcendental equations are generally referred to as nonlinear equations.

The numerical solution of nonlinear equations depends on the number of iterations required and the cost per iteration, making it the most complicated problem in scientific computations. Nonlinear equations are mostly solved by numerical iterative methods. There are several numerical iterative techniques used to find the roots of the nonlinear expressions. A system of nonlinear equations is a system of two or more equations in two or more variables involving at least one equation that is not linear. Solving nonlinear equations is an important field in numerical analysis. It is also one of the oldest and most basic problems of mathematics, engineering, and computer science applications (Karthikeyan, 2011).

Numerical analysis and computers are intimately related to each other regarding mathematical problems. With the development of computers, numerical methods have been improved for solving scientific and engineering problems. The Numerical Iterative Methods are used to find approximate solutions to such problems which are not always possible to get closed-form solutions by using algebraic processes (Grosan & Abraham, 2008).

The numerical iterative scheme for the solution of nonlinear equations includes Secant and Newton's Methods. The convergence rate of NRM is faster when compared with other numerical methods but it was demanding to take both speed and cost of convergence. For the solutions of these nonlinear equations, various iterative schemes are applied i.e. NRM and its variants (Karthikeyan, 2011). Numerical Iterative Techniques like NRM are mostly used to get the approximated solution of the nonlinear problems since it is not always possible to have the correct solution by old algebraic methods (Allame & Azad, 2012). NRM is a powerful and well-known iterative technique known to converge quadratically and can converge more rapidly than any other Numerical Iterative Method (Ehiwario & Aghamie, 2014). It requires more iteration than the Improve Iterative Method (Nazeer et al., 2016).

To get an approximated solution of a nonlinear equation is a very important task in numerical analysis. The solution of the nonlinear algebraic expressions $f(x) = 0$ is usually named NRM. This technique is a famous numerical iterative method and is frequently considered the very influential method used to solve the equation $f(x) = 0$ (Ebelechukwu et al., 2018). Some authors also improve and modify the technique NRM from various aspects.

Researchers improved the order and accuracy of NRM by applying ADM (Kang et al., 2015). The Modified Adomian Decomposition Method (MADM) is a simple Iterative method to solve nonlinear equations (Abbaoui & Cherruault, 1995). A convergence proof of ADM based on the properties of convergent series is proposed where few outcomes are deduced about the speed of convergence which allows us to compute nonlinear functional expressions (Cherruault & Adomian, 1993). For the improvement of the order of convergence, many modified methods have been suggested (Ali et al., 2015; Kou, 2007; Chun, 2007). These methods are derived by extending $f(x) = 0$ to the 3rd order and MADM techniques applied.

In order to achieve two major improvements (Aristizabal et al., 2023), the author recently added hypercomplex algebra to the traditional Newton-Raphson (NR) method for solving non-linear systems of equations: computing the Jacobian with high accuracy and computing the NR solution's sensitivities concerning any design parameter. Recently the authors studied the linear models for the prediction of the initial estimate for the nonlinear Newton-Raphson solver and suggested that this method reduces the number of iterations in the Newton-Raphson algorithm and expedites simulation time (Petrosyants et al., 2024).

Therefore, there are abundant applications where many researchers use the NRM in different aspects (Ali et al., 2015; Kou, 2007; Chun, 2007; Kang et al., 2015). Their work inspires our work, and we have tried to improve the MLADM technique further. In the present paper, our research goal is to improve NRM with the help of the MADM. The rest of the article is organized as follows.

Next section presents the analysis of the proposed research process. Then the applications of the proposed method and numerical results are considered for the nonlinear equations by NPM, FPM, Algorithm 2.1, Algorithm 2.2, and Algorithm 2.3. The final section gives a brief conclusion of the proposed study.

Analysis of Proposed Research Process

Taylor series is reachable to all researchers and is a very significant mathematical technique for nonlinear equations (He & Ji, 2019). The main importance of Taylor series is that Taylor series converts nonlinear equations into polynomial equations.

In this research, the Taylor series is considered at least up to four terms and eliminates the value of $"x"$ from the first term of the Taylor series which is mathematically written in the form of:

$$f(x_0) + (x - x_0)f'(x_0) + (x - x_0)^2 f''(x_0) + (x - x_0)^3 f'''(x_0) = 0$$

Now eliminating 'x' and considering $x_1 = N(x_0)$, we get:

$$x_1 = \frac{1}{2}(x - x_0)^2 - \frac{(x - x_0)^3}{6}\frac{f'''(x_0)}{f'(x_0)}$$

$$\text{or } N(x_0) = -\frac{1}{2}(x - x_0)^2 - \frac{(x - x_0)^3}{6}\frac{f'''(x_0)}{f'(x_0)}$$

Now applying the Adomian Decomposition technique on $A_0 = N(x_0)$ say equation 1 we will get another equation $A_1 = x_1 N'(x_0)$ say equation 2. Now the convergence series of 2 is $x = c$ and $X_{n+1} = A_n$ where $n \geq 0$. The Wazwaz generated all kinds of polynomials, non-linear polynomials are generated for all kinds of non-linearity and some of them are given as under:

$$A_0 = N(x_0) \tag{1}$$

$$A_1 = x_1 N'(x_0) \tag{2}$$

$$A_2 = x_2 N'(x_0) + \frac{1}{2}x_1^2 N''(x_0) \tag{3}$$

Where the formula of NRM is given as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},\ n = 0, 1, 2, 3, \ldots \ldots \ldots$$

Further polynomials can be developed similarly. Now putting values and then solving the above equations 1 and 2 to get Algorithm 2.1:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^2(x_n)}{2f'(x_n)}f''(x_n) + \frac{f^3(x_n)f'''(x_n)}{6f'(x_n)} \tag{4}$$

Algorithm 2.2 is given as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^2(x_n)}{2f'(x_n)}f''(x_n) + \frac{f^3(x_n)f'''(x_n)}{6f'(x_n)} + \frac{f^3(x_n)f''^2(x_n)}{2f'^5(x_n)} + \frac{5f^4(x_n)f''(x_n)f'''(x_n)}{12f'^6(x_n)} \tag{5}$$

It is noted that Algorithm 2.2 is rapidly convergent as compared to Algorithm 2.1. In this research, we will develop a new Algorithm 2.3 and will solve some different nonlinear equations on FPM, NRM, Algorithm 2.1, 2.2, and on the suggested Algorithm 2.3, in terms of analysis of the proposed Algorithm 2.3.

Developing Algorithm 2.3

Since from the Taylor series we find the value $x_1$and then take a square of x_1 and multiply x_1^2 with $N''(x_0)$ and then divide it by 2 we will get $\frac{1}{2}x_1^2 N''(x_0)$. Similarly, we will multiply x_2 with $N'(x_0)$ and add with $\frac{1}{2}x_1^2 N''(x_0)$ we will get new Algorithm 2.3 which is most rapidly convergent with some other Numerical Iterative Methods like FPM, NRM, Algorithm 2.1, and Algorithm2.2 respectively. The values of x_1, x_1^2 , $N'(x_0)$, $N''(x_0)$, are given as:

$$x_1 = \frac{f^4(x)f''^2(x)}{4f'^6(x)} + \frac{f^6(x)f'''^2(x)}{6f'^8(x)} + \frac{f^5(x)f''(x)f'''(x)}{6f'^7(x)} \quad (6)$$

$$x_1{}^2 = \frac{-f^2(x)f''(x)}{2f'^3(x)} + \frac{f^3(x)f'''(x)}{6f'^4(x)} - \frac{f^3(x)f''^2(x)}{2f'^5(x)} + \frac{5f^4(x)f''(x)f'''(x)}{12f'(x)} - \frac{f^5(x)f'''^2(x)}{12f'^7(x)} \quad (7)$$

$$N'(x_0) = \frac{-f^2(x)f'''(x)}{2f'^3(x)} - \frac{f''(x)f(x)}{f'^2(x)} + \frac{3f^2(x)f''^2(x)}{2f'^4(x)} + \frac{f^3(x)f'^v(x)}{6f'^4(x)} + \frac{f'''(x)f^2(x)}{2f'^3(x)} \quad (8)$$

$$N''(x_0) = \frac{-f^2(x)f'^v(x)}{2f'^3(x)} - \frac{f(x)f'''(x)}{f'^2(x)} - \frac{3f^2(x)f''(x)f'''(x)}{2f'^4(x)} - \frac{f''(x)}{f'(x)} - \frac{f(x)f'''(x)}{f'^2(x)} + \frac{2f''^2(x)f(x)}{f'^3(x)} + \frac{3f^2(x)f''(x)f'''(x)}{2f'^4(x)} + \frac{3f(x)f''^2(x)}{2f'^3(x)} + \frac{f^3(x)f^v(x)}{6f'^4(x)} + \frac{f^2(x)f^{iv}(x)}{2f'^3(x)} - \frac{4f^3(x)f''(x)f^{iv}(x)}{3f'^5(x)} + \frac{f(x)f'''(x)}{f'^2(x)} + \frac{f^2(x)f^{iv}(x)}{2f'^3(x)} - \frac{3f^2(x)f''(x)f'''(x)}{2f'^4(x)} - \frac{2f^3(x)f'''^2(x)}{3f'^5(x)} - x\frac{3f^2(x)f''(x)f'''(x)}{2f'^4(x)} + \frac{10f^3(x)f''^2(x)f'''(x)}{f'^6(x)} \quad (9)$$

We will get new algorithm 2.3 by putting the values of, x_1, x_1^2, $N'(x_0)$ and $N''(x_0)$, in the following equation:

$$A_2 = x_2 N'(x_0) + \frac{1}{2} x_1^2 N''(x_0)$$

We get the new Algorithm 2.3 as follows:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f^4(x)f''(x)f'''(x)}{3f'^6(x)} - \frac{3f^4(x)f''^3(x)}{8f'^7(x)} + \frac{f^5(x)f''^2(x)f'''(x)}{8f'^8(x)} - \frac{f^5(x)f''^2(x)}{8f'^9(x)} + \frac{49f^6(x)f''^3(x)f'''(x)}{24f'^{10}(x)} - \frac{29f^7(x)f''^2(x)f'''^3(x)}{24f'^{11}(x)} + \frac{f^3(x)f''(x)}{2f'^5(x)} - \frac{5f^5(x)f''(x)f^{iv}(x)}{12f'^7(x)} +$$

$$\frac{f^6(x)f'''(x)f^{iv}(x)}{36f'^8(x)}+\frac{5f^5(x)f''^2(x)f'''(x)}{12f'^3(x)}+\frac{5f^6(x)f''^3(x)f'''(x)}{8f'^5(x)}$$
$$+\frac{5f^7(x)f''(x)f'''(x)f^{iv}(x)}{72f'^5(x)}-\frac{5f^6(x)f''(x)f'''^2(x)}{12f'^9(x)}$$
$$-\frac{f^8(x)f'''^2(x)f^{iv}(x)}{18f'^{11}(x)}-\frac{f^7(x)f''^3(x)f^{iv}(x)}{6f'^{11}(x)}$$
$$+\frac{f^6(x)f''^3(x)f'''(x)}{2f'^{10}(x)}-\frac{f^6(x)f''^2(x)f^{iv}(x)}{48f'^9(x)}$$
$$+\frac{f^7(x)f''^2(x)f^{v}(x)}{48f'^{10}(x)}+\frac{5f^7(x)f''^4(x)f'''(x)}{2f'^{12}(x)}$$
$$-\frac{f^8(x)f''(x)f'''^3(x)}{8f'^{12}(x)}-\frac{f^6(x)f''^2(x)f'''^2(x)}{12f'^{11}(x)}$$
$$-\frac{f^9(x)f'''^4(x)}{18f'^{13}(x)}+\frac{f^7(x)f''^2(x)f'''^3(x)}{6f'^{11}(x)}$$
$$+\frac{3f^8(x)f''(x)f'''^2(x)}{12f'^{12}(x)}+\frac{f^9(x)f''^2(x)f^{v}(x)}{72f'^{14}(x)}$$
$$+\frac{f^8(x)f''^2(x)f^{iv}(x)}{24f'^9(x)}+\frac{f^7(x)f'''^3(x)}{2f'^{10}(x)}$$
$$+\frac{f^8(x)f''^2(x)f^{iv}(x)}{24f'^{11}(x)}-\frac{f^8(x)f''(x)f'''^3(x)}{8f'^{12}(x)}$$
$$-\frac{f^8(x)f''(x)f''^2(x)f^{iv}(x)}{6f'^{12}(x)}+\frac{f^9(x)f''^2(x)f'''^3(x)}{6f'^{14}(x)}$$
$$-\frac{f^5(x)f''(x)f'''(x)}{12f'^8(x)}+\frac{f^8(x)f''(x)f'''(x)f^{v}(x)}{72f'^{11}(x)}$$
$$+\frac{f^7(x)f''(x)f'''(x)f^{iv}(x)}{24f'^{12}(x)}$$
$$-\frac{f^8(x)f''^2(x)f'''(x)f^{iv}(x)}{18f'^{12}(x)}+\frac{f^7(x)f''^2(x)f'''^2(x)}{8f'^{12}(x)}$$
$$+\frac{f^8(x)f''^3(x)f'''^2(x)}{3f'^{13}(x)} \quad (10)$$

Applications

This section elaborates and explains the effectiveness and generalization of the proposed Algorithm 2.3 discussed thoroughly. We solved different examples of nonlinear equations or systems of nonlinear equations by different numerical methods like NRM, FPM, Algorithm 2.1, and Algorithm 2.2 and by our newly proposed Algorithm 2.3. After solving these examples, we compare the results in the form of tables to provide a clear and comprehensive comparison. The results of these different techniques are summarized in the form of tables and discussed thoroughly. By this comparison, the best iterative method is explored for each example in terms of convergence rate and iterations. To validate the proposed Algorithm for solving the nonlinear equation, or system of nonlinear equations different examples are considered.

Example 1

Consider $Sin^2 x - x^2 + 1 = 0$ with initial guess 2. Table 1 shows the behavior of the root obtained on a given nonlinear equation through different techniques on different numbers of iterations. The FPM gave the root in the fifth iteration, NRM gave the same root in the fourth iteration, and Algorithms 2.1 and 2.2 in the third and second iterations, but remarkably the proposed algorithm 2.3 converges to the same root in the first iteration. In terms of convergence speed, this suggests that Algorithm 2.3 performs better than the other approaches because it takes fewer iterations, reducing computational work and effort, which means Algorithm 2.3 is an extremely efficient way to solve nonlinear equations. Table 1 presents a detailed and unambiguous comparison of the outcomes achieved by various methodologies, as shown by the number of iterations. In addition to highlighting Algorithm 2.3's improved efficiency over FPM, NRM, and Algorithms 2.1 and 2.2, this table shows how many iterations are needed for each approach to converge to the same root.

Table 1: Comparative Results of Techniques on Example 1

N	FPM	NRM	Algorithm 2.1	Algorithm 2.2	Algorithm 2.3
1	1.3516	1.5431	1.4605	1.4287	1.4045
2	1.3974	1.4171	1.4047	1.4045	
3	1.4037	1.4046	1.4045		
4	1.4044	1.4045			
5	1.4045				

Example 2

Let us suppose $f(x) = x^3 - 10 = 0$ with $x_0 = 2$ with an initial guess. In example 2 we encounter a nonlinear equation on different numerical methods to find the same root on different numbers of iterations. We notice that the roots obtained by FPM and NRM are given after the eleventh and third iterations respectively. While the same root is given by Algorithm 2.1 and 2.2 after the second iteration. Remarkably the proposed Algorithm 2.3 converges to that root just in a single iteration. This shows that the proposed algorithm 2.3 performs significantly as compared to other iterative methods. The results obtained by different approaches are demonstrated in terms of iterations and are summarized in Table 2 to show a clear and comprehensive comparison between these iterative techniques. This table illustrates how many iterations are required for each method to converge to the same root, and also highlights the efficiency of Algorithm 2.3 over FPM, NRM, and Algorithms 2.1 and 2.2.

Table 2: Comparative Results of Techniques on Example 2

N	FPM	NRM	Algorithm 2.1	Algorithm 2.2	Algorithm 2.3
1	2.2361	2.1667	2.1524	2.1574	2.1544
2	2.1147	2.1545	2.1544	2.1544	
3	2.1476	2.1544			
4	2.1444				
5	2.1595				
6	2.1519				
7	2.1557				
8	2.1538				
9	2.1548				
10	2.1543				
11	2.1544				

Example 3

Let $f(x) = x^3 + x^2 - 2 = 0$ with$x_0 = 0.5$ with an initial guess. In the example provided above we solved a nonlinear equation adopting various numerical techniques, each of which required a different number of iterations to reach the same root. Specifically, the FPM took nine iterations NRM took four, and Algorithm 2.1 and 2.2 converged to the same root in the third iteration. Remarkably our proposed algorithm 2.3 gets the same root just in the second iteration. This indicates that Algorithm 2.3 significantly outperforms the other methods in terms of convergence speed. Algorithm 2.3 is a very efficient method for solving nonlinear equations because it requires fewer iterations, which decreases computing effort and time.

Table 3: Comparative Results of Techniques on Example 3

N	FPN	NRM	Algorithm 2.1	Algorithm 2.2	Algorithm 2.3
1	0.89443	1.1282	1.05555	0.56710	1.00021
2	1.02749	1.0115	1.0002	0.56714	1.00000
3	0.99320	1.0001	1.0000	1.0000	
4	1.00170	1.0000			
5	0.99957				
6	1.00011				
7	0.99997				
8	1.00001				

The results obtained by different approaches are demonstrated in terms of iterations are shown in Table 3 to show a clear and comprehensive comparison. This table illustrates how many iterations are required for each method to converge to the same root, and also highlights Algorithm 2.3 superior efficiency over FPM, NRM, and Algorithms 2.1 and 2.2.

Example 4

Suppose $f(x) = \ln(x) + x = 0$ with an initial value of 0.5. In the above example 4 we solved the given nonlinear equations by different numerical methods and got the different number of iterations to approach the same root. Such as FPM contributed to that root in eight iterations, NRM gave the same root in the sixth iteration, and Algorithm 2.1 and 2.2 converged to that root after the second iteration, but our Algorithm 2.3 converged just after the first iteration. This means that the suggested approach is more efficient than that of other techniques. The comparative results are presented in Table 4.

Table 4: Comparative Results of Techniques on Example 4

N	FPM	NRM	Algorithm 2.1	Algorithm 2.2	Algorithm 2.3
1	0.60653	0.57726	0.56691	0.56710	0.56714
2	0.54524	0.56522	0.56714	0.56714	
3	0.57970	0.56750			
4	0.56007	0.56708			
5	0.57117	0.56716			
6	0.56486	0.56714			
7	0.56844				
8	0.56641				
9	0.56756				
10	0.56691				
11	0.56728				
12	0.56706				

Example 5

Consider $f(x) = e^x - 5x^2 = 0$ with initial guessed at 1.5. In the above example, 5 we solved the given nonlinear equation by different numerical methods and got the same result in different number of iterations. Such as FPM provides the root in eight iterations, NRM gives the same root in the third iteration, while Algorithm 2.1 and 2.2 converge to that root in the second iteration, but the proposed Algorithm 2.3 converges to the same root in the very first iteration to show the efficiency of the proposed method. To provide a clear and comprehensive comparison, the results of these different techniques are summarized in Table 5. The table highlights the number of iterations each method required to converge to the same root, to underscore the enhanced efficiency of algorithm 2.3 compared the obtained results with other iterative methods FPM, NRM, Algorithm 2.1, and Algorithm 2.2.

Table 5: Comparative Results of Techniques on Example 5.

N	FPM	NRM	Algorithm 2.1	Algorithm 2.2	Algorithm 2.3
1	0.57423	0.61898	0.60148	0.60660	0.60527
2	0.59595	0.60544	0.60527	0.60527	
3	0.60245	0.60527			
4	0.60442				
5	0.60501				
6	0.60519				
7	0.60524				
8	0.60526				

Conclusion

Several nonlinear and systems of nonlinear expressions are estimated via enhancements in NRM. The results are presented in the form of several iterations for converging to an exact solution. In this research, we develop a new algorithm with the help of MADM called algorithm 2.3. Through this new algorithm, we investigate a system of nonlinear equations and get better results as compared with other algorithms in terms of iterations. The obtained results of Algorithm 2.3 are compared with the generated results of other methods like NRM and FPM. The existing enhancements in NRM, Algorithms 2.1 and 2.2 are also comparatively analyzed. The solution of different examples shows that Algorithm 2.3 is performing very well and rapidly converging as compared to FPM, NRM, Algorithms 2.1 and 2.2. It takes less time to get accurate results in a very few iterations in solving nonlinear problems. It is worth considering in the future, the terms involved in new Algorithm 2.3 need to be reduced further. Similarly, Algorithm 2.3 needs to validate other nonlinear mathematical problems. This research can be extended to numerical analysis, an area of mathematics and computer sciences that creates, analyzes, and implements algorithms for achieving numerical results for problems concerning continuous variables.

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